Section 6.4 Sum and Difference Formulas

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

ex: Find exact value of $\sin 75^\circ$

$$= \sin(30^\circ+45^\circ) = \sin 30^\circ\cos 45^\circ + \cos 30^\circ\sin 45^\circ$$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

ex: $\tan\frac{\pi}{12} = \tan 15^\circ = \tan(60^\circ - 45^\circ)$

$\frac{\pi}{12} \cdot \frac{180}{\pi}$

$45^\circ - 30^\circ$

$60^\circ - 45^\circ$

$\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$= \frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \cdot \tan 45^\circ}$

$= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$\sqrt{3}-1$ is conjugate of $\sqrt{3}+1$

$= \frac{3 - \sqrt{3} - \sqrt{3} + 1}{3 - \sqrt{3} + \sqrt{3} - 1}$

$= \frac{4 - 2\sqrt{3}}{2}$

$= \frac{2(2 - \sqrt{3})}{2}$

$= 2 - \sqrt{3}$

Ex: Given $\tan\alpha = \frac{3}{4}$ ($y=3$, $x=4$), α in QI

For α, $x=4$, $y=3$, $r=5$

$$\sin\alpha = \frac{y}{r} = \frac{3}{5}$$

$$\cos\alpha = \frac{x}{r} = \frac{4}{5}$$

$\sin\beta = \frac{7}{25}$ $\quad 90^\circ < \beta < 180^\circ$ (QII)

For β, $y=7$, $r=25$

$x=-24$

$$\sin\beta = \frac{7}{25}$$

$$\cos\beta = \frac{-24}{25}$$

Find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \frac{3}{5}\cdot\frac{-24}{25} + \frac{4}{5}\cdot\frac{7}{25} = \frac{-72}{125} + \frac{28}{125} = -\frac{44}{125} = \frac{y}{r}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \frac{4}{5}\cdot\frac{-24}{25} - \frac{3}{5}\cdot\frac{7}{25} = \frac{-96}{125} - \frac{21}{125} = -\frac{117}{125} = \frac{x}{r}$$

$$\tan(\alpha+\beta) = \frac{y}{x} = \frac{-44}{-117} = \frac{44}{117}$$

What Quadrant is $\alpha+\beta$ in: QIII

44, 117, 125 is a Pythagorean Triple

$$44^2 + 117^2 = 125^2$$

$$1936 + 13689 = 15625$$

Assignment

1) Find $\sin(\alpha-\beta)$, $\cos(\alpha-\beta)$, $\tan(\alpha-\beta)$ and what Quadrant $\alpha-\beta$ is in if:

$\sin\alpha = \frac{8}{17}$, $0<\alpha<\frac{\pi}{2}$ $\quad\quad$ $\cos\beta = \frac{40}{41}$, $\frac{3\pi}{2}<\beta<2\pi$

In 2-4 use sum or difference formulas:

2) $\cos 165^\circ$

3) $\sin\frac{19\pi}{12}$

4) $\tan\frac{5\pi}{12}$