

$$\frac{\sec \theta}{\csc \theta} + \frac{\sin \theta}{\cos \theta} = 2 \tan \theta$$

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$$\frac{\sec \theta}{\csc \theta} + \frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{\cos \theta}}{\frac{1}{\sin \theta}} + \tan \theta$$

$$= \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1} + \tan \theta$$

$$= \frac{\sin \theta}{\cos \theta} + \tan \theta$$

$$= \tan \theta + \tan \theta$$

$$= 2 \tan \theta$$

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$$\sec \theta - \cos \theta - \sin \theta \tan \theta = 0$$

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$$\sec \theta - \cos \theta - \sin \theta \tan \theta = \frac{1}{\cos \theta} - \frac{\cos \theta}{1} - \frac{\sin \theta}{1} \cdot \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta} - \frac{\sin^2 \theta}{\cos \theta} - \cos \theta$$

$$= \frac{1 - \sin^2 \theta}{\cos \theta} - \cos \theta$$

$$= \frac{\cos^2 \theta}{\cos \theta} - \cos \theta$$

Establish identity

$$3 \sin^2 \theta + 4 \cos^2 \theta = 3 + \cos^2 \theta$$

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$$\begin{aligned} 3 \sin^2 \theta + 4 \cos^2 \theta &= 3(1 - \cos^2 \theta) + 4 \cos^2 \theta \\ &= 3 - 3 \cos^2 \theta + 4 \cos^2 \theta \\ &= 3 + \cos^2 \theta \end{aligned}$$

$$-3x + 4x = x$$

$$= \cos \theta - \cos \theta$$
$$= 0$$

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$$1 - \frac{\cos^2 \theta}{1 + \sin \theta} = \sin \theta$$

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$$A^2 - B^2 = (A - B)(A + B)$$

$$1 - \frac{\cos^2 \theta}{1 + \sin \theta} = 1 - \frac{1 - \sin^2 \theta}{1 + \sin \theta}$$

$$= 1 - \frac{(1 - \sin \theta)(1 + \sin \theta)}{1 + \sin \theta}$$

$$= 1 - 1 + \sin \theta$$

$$= \sin \theta$$

$$1 - \frac{\sin^2 \theta}{1 - \cos \theta} = -\cos \theta$$

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$$1 - \frac{\sin^2 \theta}{1 - \cos \theta} = 1 - \frac{1 - \cos^2 \theta}{1 - \cos \theta}$$

$$= 1 - \frac{(1 - \cos \theta)(1 + \cos \theta)}{1 - \cos \theta}$$

$$= 1 - 1 - \cos \theta$$

$$= -\cos \theta$$

$$\frac{\sin\theta + \cos\theta}{\sin\theta} - \frac{\cos\theta - \sin\theta}{\cos\theta} = \sec\theta \csc\theta$$

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$$\frac{\sin\theta + \cos\theta}{\sin\theta} - \frac{\cos\theta - \sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta} \frac{(\sin\theta + \cos\theta)}{\sin\theta}$$

$$\text{LCD} = \sin\theta \cos\theta \quad - \frac{\sin\theta}{\sin\theta} \frac{(\cos\theta - \sin\theta)}{\cos\theta}$$

$$= \frac{\cancel{\cos\theta \sin\theta} + \cos^2\theta - \cancel{\cos\theta \sin\theta} + \sin^2\theta}{\sin\theta \cos\theta}$$

$$= \frac{1}{\cos\theta \sin\theta}$$

$$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$$

$$= \sec\theta \cdot \csc\theta$$

$$(2a \sin\theta \cos\theta)^2 + a^2 (\cos^2\theta - \sin^2\theta)^2 = a^2$$

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$$(2a \sin\theta \cos\theta)^2 + a^2 (\cos^2\theta - \sin^2\theta)^2$$

$$= \underline{4a^2 \sin^2\theta \cos^2\theta} + \underline{a^2 (\cos^4\theta - 2\sin^2\theta \cos^2\theta + \sin^4\theta)}$$

$$= a^2 [4\sin^2\theta \cos^2\theta + \cos^4\theta - 2\sin^2\theta \cos^2\theta + \sin^4\theta]$$

$$= a^2 \left[ \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \sin^4 \theta \right]$$

$$= a^2 (\cos^2 \theta + \sin^2 \theta)^2$$

$$= a^2 \cdot 1^2$$

$$= a^2$$

$$\frac{x - y}{z} = \frac{x}{z} - \frac{y}{z}$$

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$$\frac{\sec \theta - \csc \theta}{\sec \theta \csc \theta} = \sin \theta - \cos \theta$$

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$$\frac{\sec \theta - \csc \theta}{\sec \theta \csc \theta} = \frac{\cancel{\sec \theta}}{\cancel{\sec \theta} \csc \theta} - \frac{\cancel{\csc \theta}}{\sec \theta \cancel{\csc \theta}}$$

$$= \frac{1}{\csc \theta} - \frac{1}{\sec \theta}$$

$$= \frac{1}{\frac{1}{\sin \theta}} - \frac{1}{\frac{1}{\cos \theta}}$$

$$= \sin \theta - \cos \theta$$