WARMUP

1) Given $f(x) = x^2 - 3x$, calculate

$$\frac{f(3+h) - f(3)}{h} = \frac{\left[\overbrace{(3+h)^2}^{(3+h)(3+h)} - 3(3+h)\right] - \left[3^2 - 3\cdot 3\right]}{h}$$

$$= \frac{\cancel{9} + 6h + h^2 - \cancel{9} - 3h - 0}{h}$$

$$= \frac{3h + h^2}{h}$$

$$= \frac{\cancel{h}(3+h)}{\cancel{h}}$$

$$= 3 + h$$

2) $\lim\limits_{h \to 0} \dfrac{\sqrt{10+h} - \sqrt{10}}{h}$

$$\lim_{h \to 0} \frac{(\sqrt{10+h} - \sqrt{10})}{h} \frac{(\sqrt{10+h} + \sqrt{10})}{(\sqrt{10+h} + \sqrt{10})}$$

$$\lim_{h \to 0} \frac{\overset{1}{\cancel{10} + \cancel{h}} - \cancel{10}}{\cancel{h}(\sqrt{10+h} + \sqrt{10})}$$

$$\frac{1}{\sqrt{10} + \sqrt{10}} = \frac{1}{2\sqrt{10}}$$

Section 2.3 The Derivative at a Point

Avg Rate of change on $[a, a+h]$ $= \dfrac{f(a+h) - f(a)}{h}$

The **derivative of f at a**, written $f'(a)$ ("f prime of a") is defined as

$$\begin{matrix}\text{Rate of}\\ \text{change of}\\ f \text{ at } a\end{matrix} = \begin{matrix}\text{Slope of tangent}\\ \text{line at } (a, f(a))\end{matrix} = f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

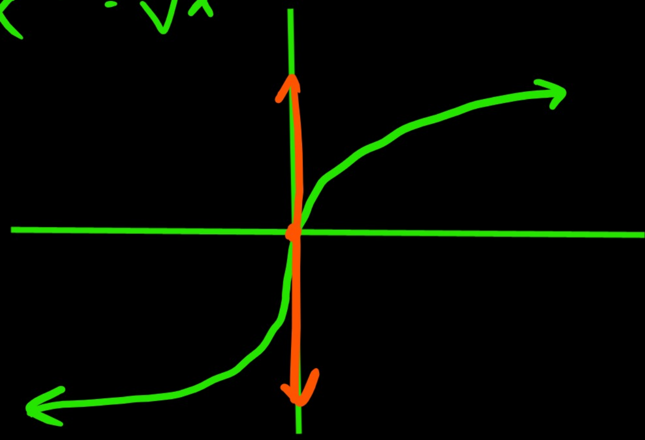
* Rate of Change $\Longleftrightarrow$ Slope of Tangent Line $\Longleftrightarrow$ Derivative

When you are asked to calculate $f'(a)$ using the **definition of derivative** you must use

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If $\displaystyle\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ exists, we say f is **differentiable** at $x = a$. The process of calculating the derivative is called **differentiation**.

ex: $f(x) = x^{1/3} = \sqrt[3]{x}$



Not differentiable at $x=0$ since there's a vertical tangent so slope is undefined.

ex: Calculate $f'(-2)$ if $f(x) = 2x^2 + 3x - 4$ using definition of derivation.

$$f'(-2) = \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h}$$

$$= \lim_{h \to 0} \frac{\overbrace{[2(-2+h)^2 + 3(-2+h) - 4]}^{f(-2+h)} - \overbrace{[2(-2)^2 + 3(-2) - 4]}^{f(-2)}}{h}$$

$$= \lim_{h \to 0} \frac{[2(4-4h+h^2) - 6 + 3h - 4] - [\overbrace{8-6-4}^{-2}]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{8} - 8h + 2h^2 - \cancel{6} + 3h - \cancel{4} + \cancel{2}}{h}$$

$$= \lim_{h \to 0} \frac{2h^2 - 5h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2h-5)}{\cancel{h}}$$

$$= 2 \cdot 0 - 5 = \boxed{-5}$$

ex: $f'(9)$ if $f(x) = \sqrt{x}$

$$f'(9) = \lim_{h \to 0} \frac{f(9+h) - f(9)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{9+h} - \sqrt{9}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{9+h} - 3)}{h} \frac{(\sqrt{9+h} + 3)}{(\sqrt{9+h} + 3)}$$

$$= \lim_{h \to 0} \frac{\cancel{9} + \cancel{h}^{\,1} - \cancel{9}}{\cancel{h}(\sqrt{9+h} + 3)}$$

$$= \frac{1}{\sqrt{9} + 3} = \frac{1}{3+3} = \frac{1}{6}$$

Assignment

Using definition of derivative find:

1) $f'(-3)$ if $f(x) = 5x^2 - 4$

2) $f'(16)$ if $f(x) = 2\sqrt{x}$