

Section 7.3 Law of Cosines

Use when SAS or SSS

For SAS

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

For SSS:

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos \beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

ex: Solve the triangle

$$a = 7.2$$

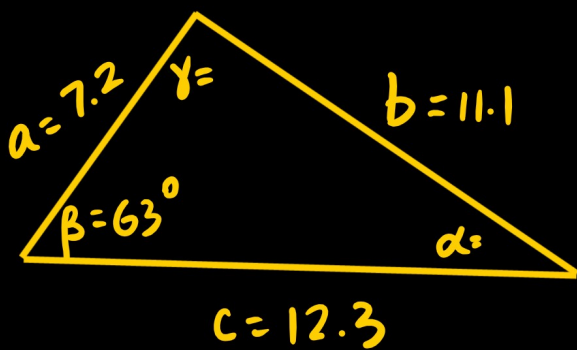
$$c = 12.3$$

$$\beta = 63^\circ$$

$$b = \underline{11.1}$$

$$\alpha = \underline{36.1^\circ}$$

$$\gamma = \underline{80.9^\circ}$$



$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$b^2 = 7.2^2 + 12.3^2 - 2 \cdot 7.2 \cdot 12.3 \cos 63^\circ$$

$$\sqrt{b^2} = \sqrt{122.72}$$

$$b = 11.1$$

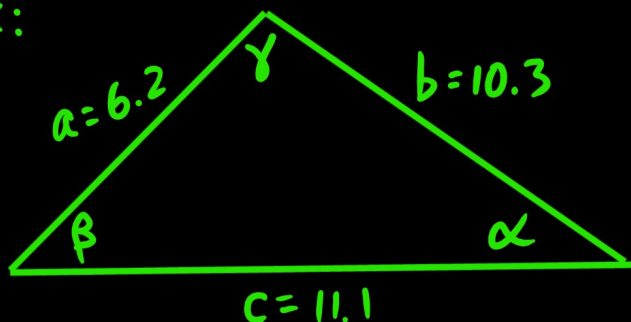
Find biggest remaining angle: $\frac{\sin \gamma}{12.3} = \frac{\sin 63^\circ}{11.1}$

$$\alpha = 180 - 63 - 80.9 = 36.1^\circ$$

$$\sin \gamma = \frac{12.3 \sin 63^\circ}{11.1} = .9873$$

$$\gamma = \sin^{-1}(.9873) = 80.9^\circ$$

ex:



$$\alpha = \frac{33.4^\circ}{\leftarrow 180 - 80.4 - 66.2} = 33.4^\circ$$

$$\beta = 66.2^\circ$$

$$\gamma = 80.4^\circ$$

In SSS case, always find the biggest angle first.

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab} = \frac{(6.2^2 + 10.3^2 - 11.1^2)}{(2 \cdot 6.2 \cdot 10.3)} = 0.1669$$

$$\gamma = \cos^{-1}(.1669) = 80.4^\circ$$

$$\cos \beta = \frac{a^2 + c^2 - b^2}{2ac} = \frac{(6.2^2 + 11.1^2 - 10.3^2)}{(2 \cdot 6.2 \cdot 11.1)} = 0.4037$$

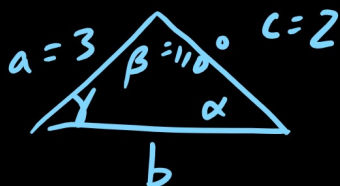
$$\beta = \cos^{-1}(.4037) = 66.2^\circ$$

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13) $a=3$

$b=2$

$\gamma = 110^\circ$



$b = 4.1$

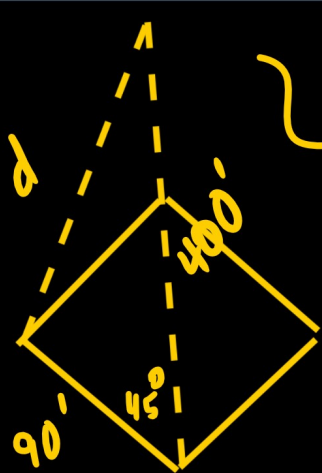
$\alpha = 43.9^\circ$

$\gamma = 26.1^\circ$

$$b^2 = 3^2 + 2^2 - 2 \cdot 3 \cdot 2 \cos 110^\circ$$

$$b^2 = 17.10$$

33)



$$b = 4.1$$

$$\cos \alpha = \frac{(4.1^2 + 2^2 - 3^2)}{(2 \cdot 4.1 \cdot 2)}$$

$$\cos \alpha = .7201$$

$$\alpha = 43.9^\circ$$

$$d^2 = 90^2 + 400^2 - 2 \cdot 90 \cdot 400 \cos 45^\circ$$