Section 2.2 Properties of Limits

Theorem 1.2 Properties of Limits

1) If b is a constant, $\lim\limits_{x\to c}(b f(x)) = b(\lim\limits_{x\to c} f(x))$

2) $\lim\limits_{x\to c}(f(x) \pm g(x)) = \lim\limits_{x\to c} f(x) \pm \lim\limits_{x\to c} g(x)$

3) $\lim\limits_{x\to c}(f(x) \cdot g(x)) = \lim\limits_{x\to c} f(x) \cdot \lim\limits_{x\to c} g(x)$

4) $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$

5) $\lim\limits_{x\to c} k = k$

6) $\lim\limits_{x\to c} x = c$

ex: $\lim\limits_{x\to 4} \dfrac{x^2-3}{2x+1} = \dfrac{\lim\limits_{x\to 4}(x^2-3)}{\lim\limits_{x\to 4}(2x+1)}$

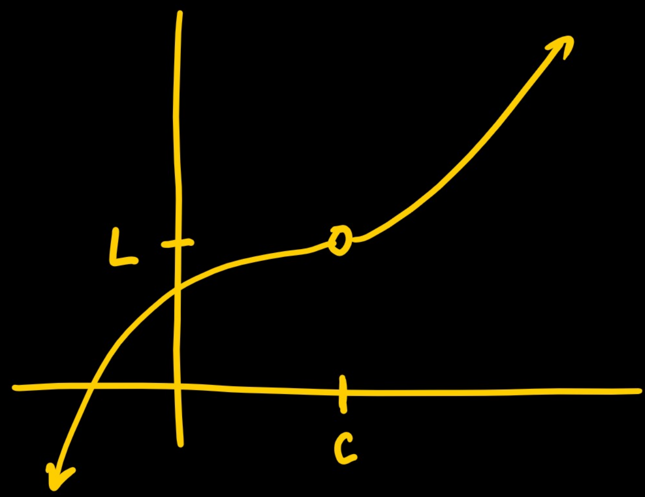
$$= \frac{\lim\limits_{x\to 4} x^2 - \lim\limits_{x\to 4} 3}{\lim\limits_{x\to 4} 2x + \lim\limits_{x\to 4} 1}$$

$$= \frac{\lim\limits_{x\to 4} x \cdot \lim\limits_{x\to 4} x - 3}{2\lim\limits_{x\to 4} x + 1}$$

$$= \frac{4 \cdot 4 - 3}{2 \cdot 4 + 1}$$

$$= \frac{13}{9}$$

Consider the graph of $f(x)$:



We say $\lim_{x \to c} f(x) = L$

limit as x approaches c of $f(x)$ is L.

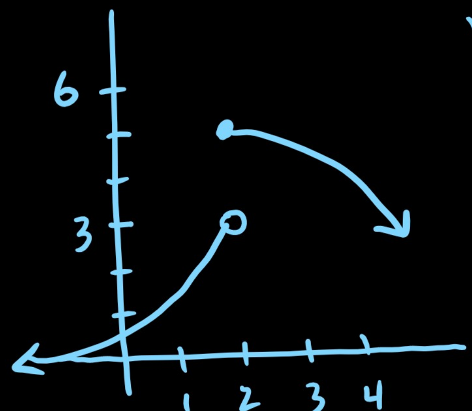
As x gets close to c, y gets close to L.

Note that $f(c) \neq L$ since there's a hole in graph

We can use a table to evaluate a limit:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Left-hand and Right-hand Limits



Left-hand limit $\lim_{x \to 2^-} f(x) = 3$

limit as x approaches 2 from the left

Right-hand limit $\lim_{x \to 2^+} f(x) = 5$

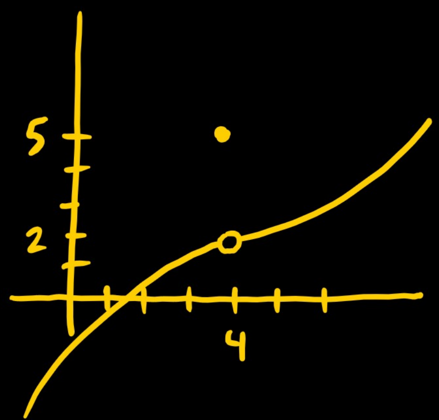
limit as x approaches 2 from the right

In this case $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

we say $\underbrace{\lim_{x \to 2} f(x)}$ does not exist

2-sided limit $\Rightarrow$ to exist, left-hand and right-hand limits must exist and be equal.

ex:



$\lim_{x \to 4^-} f(x) = 2$

$\lim_{x \to 4^+} f(x) = 2$

$\lim_{x \to 4} f(x) = 2$

$f(4) = 5$

ex: $\lim_{x \to 5} \frac{x^2-16}{x+4} = \frac{5^2-16}{5+4} = \frac{25-16}{9} = \frac{9}{9} = 1$

ex: $\lim_{x \to -4} \frac{x^2-16}{x+4} = \lim_{x \to -4} \frac{\cancel{(x+4)}(x-4)}{\cancel{x+4}} = \lim_{x \to -4} (x-4)$

$= -4-4$

$= -8$

ex: $\lim_{h \to 0} \dfrac{\left(\frac{5}{4+h} - \frac{5}{4}\right)}{h}$ $\quad \dfrac{4(4+h)}{4(4+h)}$

$\lim_{h \to 0} \dfrac{20 - 5(4+h)}{h(4+h)4}$

LCD $= 4(4+h)$

$= \lim_{h \to 0} \dfrac{\frac{5}{\cancel{4+h}} \cdot \frac{4\cancel{(4+h)}}{1} - \frac{5}{\cancel{4}} \cdot \frac{\cancel{4}(4+h)}{1}}{h \cdot 4(4+h)}$

$= \lim_{h \to 0} \dfrac{20 - 5(4+h)}{h \cdot 4(4+h)}$

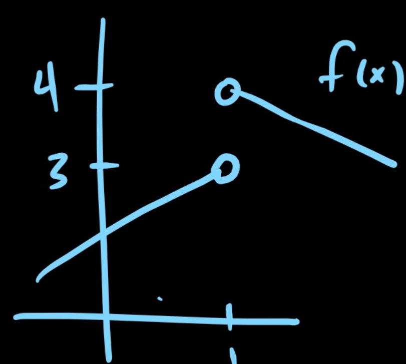
$= \lim_{h \to 0} \dfrac{\cancel{20} - \cancel{20} - 5h}{h \cdot 4(4+h)}$

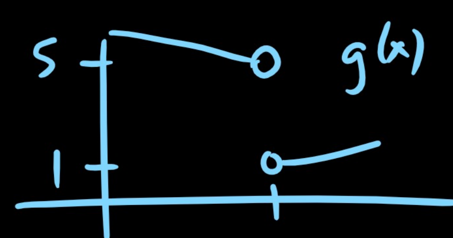
$= \lim_{h \to 0} \dfrac{-5\cancel{h}}{\cancel{h} \cdot 4(4+h)}$

$= \dfrac{-5}{4(4+0)}$

$= -\dfrac{5}{16}$

$\boxed{\text{p68 } 1, 2, 15, 16, 18}$





$\lim_{x \to 1^-} \left(f(x) + g(x)\right)$

$3 + 5 = 8$

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$\sqrt{10}\,\sqrt{10} = \sqrt{100} = 10$

$\sqrt{8}\,\sqrt{8} = \sqrt{64} = 8$

$\sqrt{x}\,\sqrt{x} = x$

18) $\lim_{h\to 0} \frac{(\sqrt{4+h}-2)}{h} \frac{(\sqrt{4+h}+2)}{(\sqrt{4+h}+2)} = \lim_{h\to 0} \frac{(4+h)-4}{h(\sqrt{4+h}+2)}$

$= \lim_{h\to 0} \frac{\cancel{h}\,1}{\cancel{h}(\sqrt{4+h}+2)}$

$= \frac{1}{\sqrt{4+0}+2} = \frac{1}{2+2} = \frac{1}{4}$

16) $\lim_{h\to 0} \frac{\frac{1}{1+h} - 1}{h}$

$\lim_{h\to 0} \frac{1-(1+h)}{h(1+h)1}$

$\lim_{h\to 0} \frac{1-1-h}{h(1+h)} = \lim_{h\to 0} \frac{\overset{-1}{\cancel{-h}}}{\cancel{h}(1+h)} = \lim_{h\to 0} \frac{-1}{1+h} = -1$