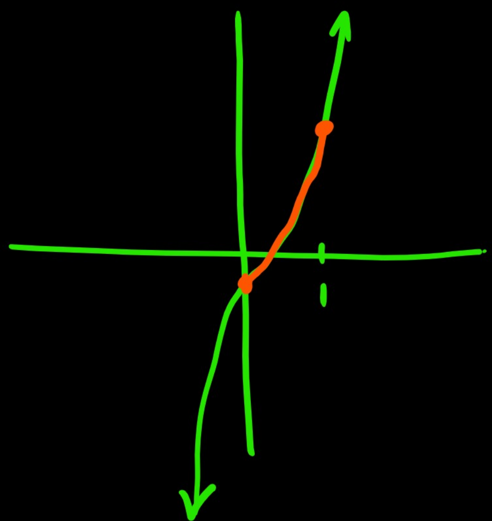
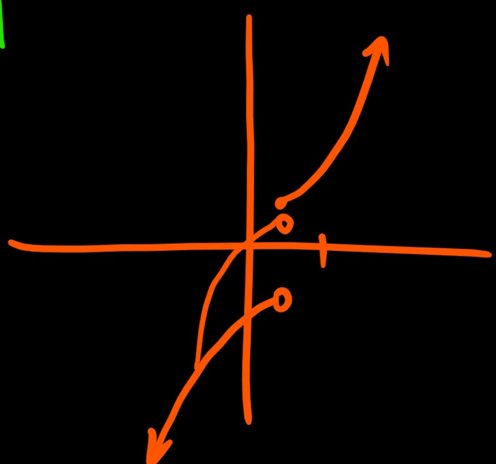
# Section 1.7 Continuity

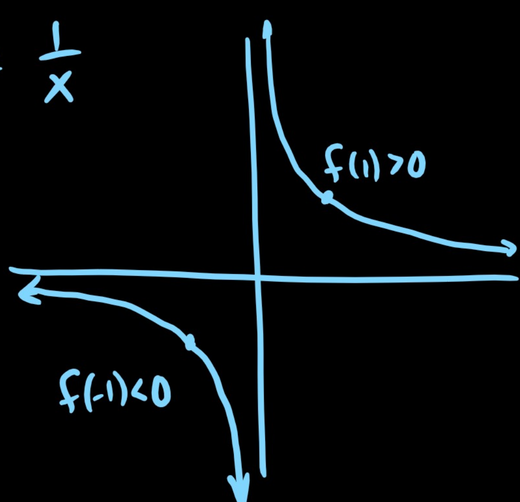
A graph is <u>continuous</u> on an interval if it has no breaks (like asymptotes), jumps, or holes on that interval.




$f(x) = 3x^3 - x^2 + 2x - 1$ is continuous everywhere.
Note that this continuity guarantees a zero (x-int) between 0 and 1.



$f(x) = \frac{1}{x}$




$f(-1) < 0$ and $f(1) > 0$

But since there's a break (not continuous) at $x = 0$, we are <u>not</u> guaranteed an x-intercept between $-1$ and $1$

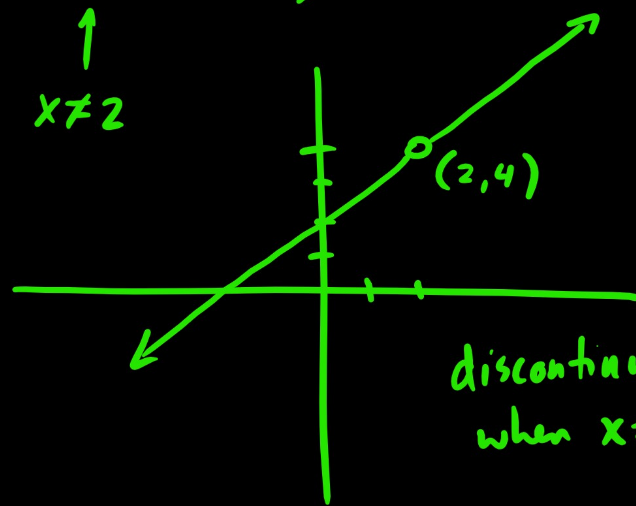
We say there's a discontinuity at $x = 0$ since there's a vertical asymptote at $x = 0$.

Another example of a discontinuity is when the graph has a hole.

ex: $f(x) = \frac{x^2-4}{x-2} = \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = x+2, \quad x \neq 2$

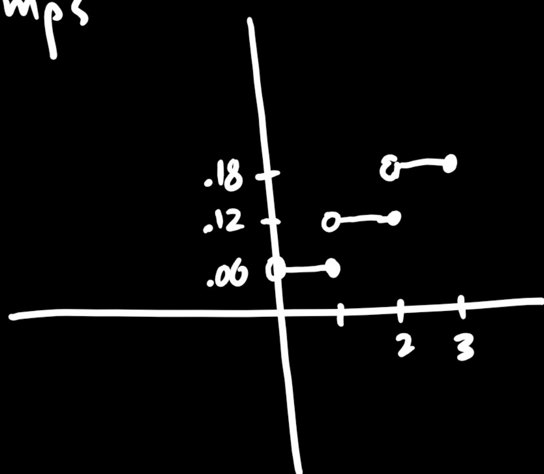
$\uparrow$

$x \neq 2$




discontinuity when $x = 2$

Step functions have many discontinuities which are jumps




"Jump" discontinuity

Some functions are continuous everywhere (for all reals)

- linear, polynominal, exponentials, sines, and cosines.

The function $y = \log x$ is continuous for $x > 0$

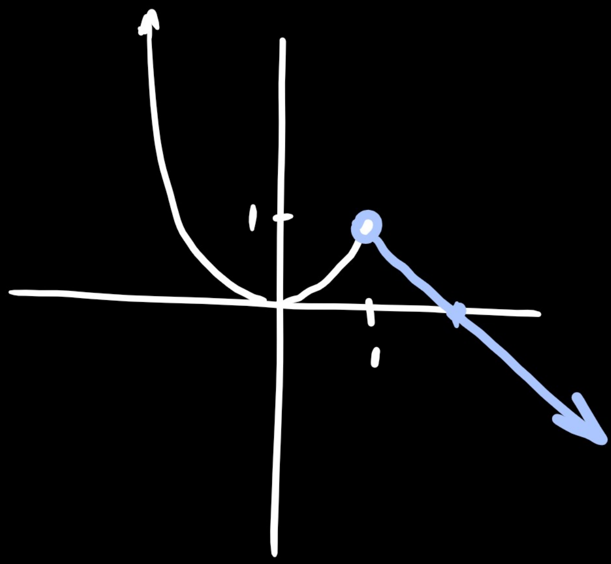
A rational function like $f(x) = \frac{x^3-27}{x+4}$ is continuous except where denominator $= 0$. So this function is continuous except when $x = -4$.

ex: Is $f(x) = 2x + x^{-1}$ continuous on $[-1, 1]$?

$$f(x) = 2x + \frac{1}{x} \Rightarrow x \neq 0$$

no since $x = 0$ makes $f(x)$ undefined

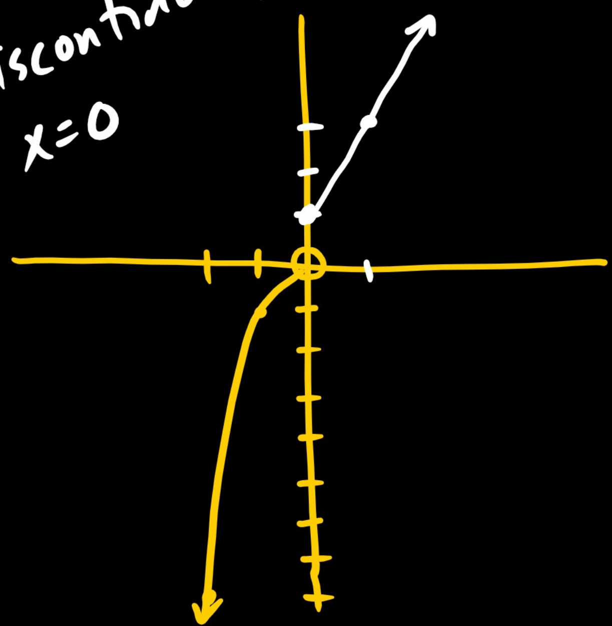
ex: Graph $f(x) = \begin{cases} x^2, & x \leq 1 \\ 2-x, & x > 1 \end{cases}$ "piecewise defined"




continuous everywhere

ex: Graph $f(x) = \begin{cases} x^3, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$

jump discontinuity at $x=0$



| x | y |
|---|---|
| -2 | -8 |
| -1 | -1 |
| 0 | 0 ← open |
| 0 | 1 ← closed |
| 1 | 3 |
| 2 | 5 |

ex: Find k so that $f(x) = \begin{cases} kx^2 & x < -1 \\ -x+2 & x \geq -1 \end{cases}$ is continuous everywhere

$3x^2 \Rightarrow 3(-1)^2 = 3$

$-x+2 \Rightarrow -(-1)+2 = 3$

$4x^2$

2 graphs must meet when $x = -1$

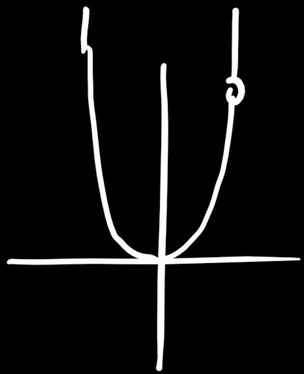
$kx^2 = -x+2$ when $x = -1$

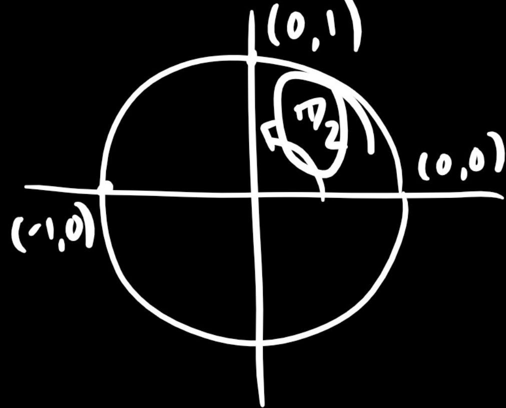
$k(-1)^2 = -(-1)+2$

$k = 1+2$

$k = 3$

p47-48
1-9 odd,
13, 15, 18






$$\frac{1}{\cos x} = \frac{1}{\cos\frac{\pi}{2}}$$

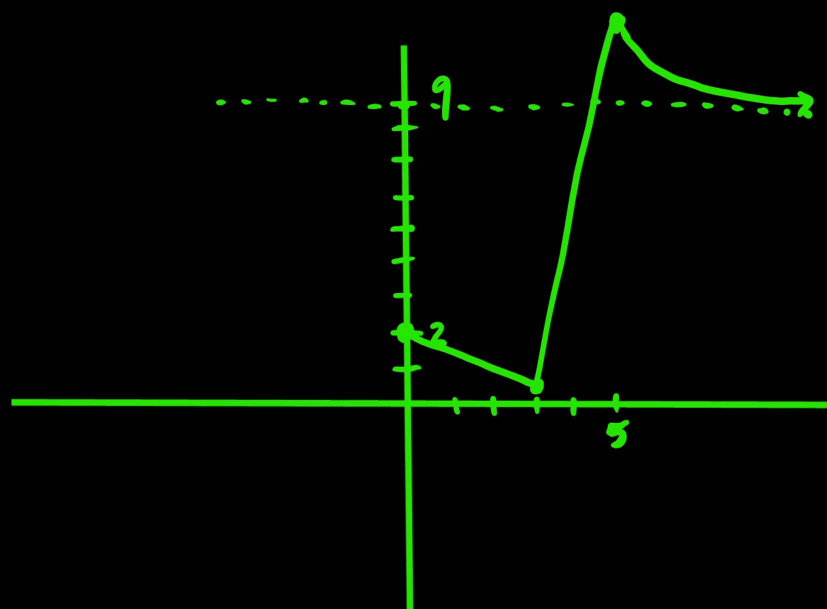
15) $$f(x) = \begin{cases} \dfrac{5x^3-10x^2}{x-2} & x \neq 2 \\ k & x = 2 \end{cases}$$

$$\frac{5x^2\cancel{(x-2)}}{\cancel{x-2}} = 5x^2$$

$$5 \cdot 2^2 = k$$

$$20 = k$$

18)




b) no