WARMUP

Find domain and V.As. of each

1) $f(x) = \frac{3x}{x^2 - 81}$

$= \frac{3x}{(x+9)(x-9)}$

$x^2 - 81 = 0$

$(x+9)(x-9) = 0$

$x = -9 \quad x = 9$

$D = (-\infty, -9) \cup (-9, 9) \cup (9, \infty)$

V.A.s $x = -9$, $x = 9$

2) $g(x) = \frac{6x - x^2}{x^2 + 5x}$

$\frac{\cancel{x}(6-x)}{\cancel{x}(x+5)}$

$D = (-\infty, -5) \cup (-5, 0) \cup (0, \infty)$

V.A. $x = -5$

3) $h(x) = \frac{x^2 - 7x + 6}{x^2 - 36}$

Section 3.4 Continued

A **horizontal asymptote** (H.A.) is a horizontal line the graph approaches as $x \to \infty$ or $x \to -\infty$

To find the H.A., we compare the degrees of the numerator and denominator.

1) If degree of top < degree of bottom, then $y = 0$ (or the x-axis) is the H.A.
2) If degree of top = degree of bottom, then y = ratio of leading coefficients is HA.

3) If degree of top > degree of bottom, then there is no H.A.

However, if degree of top = 1 + degree of bottom, then we have an oblique asymptote, O.A. (slant asymptote)

ex: Find H.A. of

a) $f(x) = \dfrac{3x^4 + 7x - 5}{8x^4 - 13x^2 + 7}$

$\left.\begin{array}{l}\text{deg of top} = 4 \\ \text{deg of bot} = 4\end{array}\right\}$ equal

$y = \dfrac{3}{8}$ is HA.

b) $f(x) = \dfrac{x}{x^2 + 7x - 6}$

$\left.\begin{array}{l}\text{deg of top} = 1 \\ \text{deg of bot} = 2\end{array}\right\}$ top < bot

$y = 0$ is HA.

To find the oblique asymptote (O.A.) which is a line in $y = mx + b$ form, we long divide the numerator by the denominator. Then $y =$ quotient is the O.A.

$350 \div 8$

$$\begin{array}{r} 43 \\ 8\,\overline{)\,350} \\ \underline{32\downarrow} \\ 30 \\ \underline{24} \\ 6 \end{array}$$

← quotient

ex: $f(x) = \dfrac{2x^2 - 5x + 4}{x + 7}$ ← deg of top = 2, ← deg of bot = 1

top = 1 + bot so O.A.

$$\begin{array}{r} 2x \quad -19 \\ x+7\,\overline{)\,\cancel{2x^2} - 5x + 4} \\ 2x(x+7) \longrightarrow \quad \underline{-(\cancel{2x^2} + 14x) \quad \downarrow} \end{array}$$

$$\begin{array}{r} -19x+4 \\ -19(x+7) \to \underline{-(-19x-133)} \\ 137 \end{array}$$

so $y = 2x - 19$ is O.A.

Find H.A. (or O.A.) of

1) $f(x) = \dfrac{3x^2 - 7x + 4}{-7x^2 + 5}$

2) $f(x) = \dfrac{5x^2 + 7}{3x^3 + 4x - 1}$

3) $f(x) = \dfrac{5x^2 + x - 7}{x - 4}$

BONUS:

$$\left(\frac{5x^2+15x-50}{2x^2+13x-7} \cdot \frac{3x^2+26x+35}{x^2+2x-8}\right) \div \left(\frac{x^2+2x+15}{8x^2-10x+3} \cdot \frac{12x^2+11x-15}{x^2+9x+20}\right)$$