WARMUP

Expand $(x+3y)^4$

$$= 1x^4(3y)^0 + 4x^3(3y)^1 + 6x^2\overset{9y^2}{(3y)^2} + 4x^1\overset{27y^3}{(3y)^3} + 1x^0(3y)^4$$

$$= x^4 + 12x^3y + 54x^2y^2 + 108xy^3 + 81y^4$$

Factorial

$0! = 1$

If $n \neq 0$, $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$

$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

$$_5C_3 = \binom{5}{3}$$

$$\frac{5!}{(5-3)!\,3!} = \frac{5\cdot \overset{2}{\cancel{4}}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{2}\cdot 1\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}} = 10$$

A B C D E

ABC BCD
ABD BCE
ABE BDE
ACD CDE
ACE
ADE

$$\binom{4}{0} = \frac{4!}{(4-0)!\,0!} = \frac{4!}{4!} = 1$$

$$\binom{4}{1} = 4$$

$$\binom{4}{2} = 6$$

$$\binom{4}{3} = 4$$

$$\binom{4}{4} = 1$$

1 4 6 4 1
row in
Pascal's Δ

on calculator $\binom{8}{3} = {}_8C_3$

8 MATH PRB nCr 3 enter

ex: Find the 3rd term of $(6x+7y)^4$

$$1(6x)^4(7y)^0 + 4(6x)^3(7y)^1 + \boxed{6(6x)^2(7y)^2} + 4 \qquad + 1$$

$$\binom{4}{0} \qquad \binom{4}{1} \qquad \binom{4}{2} \qquad \binom{4}{3} \qquad \binom{4}{4}$$

3rd term

$$6 \cdot 36x^2 \cdot 49y^2$$

$$10{,}584x^2y^2$$

j^{th} term of $(a+b)^n$

$$\binom{n}{j-1} a^{n-(j-1)} b^{j-1}$$

$n = 8$

6^{th} term of $(x+3y)^8$

$j = 6$

$$\binom{8}{5} x^{8-5} (3y)^5 = 56x^3 \cdot 243y^5$$

$$= 13{,}608x^3y^5$$

$6-1$

${}_8C_5 = 56$

8 MATH PRB nCr 5 enter

3rd term of $(2x-5y)^{11}$

$2x + (-5y)$

$$\binom{11}{2}(2x)^9(-5y)^2 = 55 \cdot 512x^9 \cdot 25y^2$$

$= 704{,}000x^9y^2$

Assignment 10/23

1) 3rd term of $(3x-2)^9$

2) 10th term of $(x+y)^{15}$

3) 8th term of $(5x-6y)^7$

4) 2nd term of $(6x^2+5y^2)^{10}$

5) 4th term of $(x^3-3y^2)^6$

$\binom{10}{1}(6x^2)^9(5y^2)^1$

$10 \cdot 10077696x^{18} \cdot 5y^2$

$503{,}884{,}800x^{18}y^2$

$(4x^2+3y)^{51}$

$\binom{51}{24}(4x^2)^{27} \cdot (3y)^{24}$

$229{,}591{,}913{,}401{,}900 \cdot 18{,}014{,}398{,}509{,}481{,}984$

$\cdot 282{,}429{,}536{,}481\,x^{54}y^{24}$

$1{,}168{,}117{,}328{,}566{,}079{,}160{,}468{,}764{,}785{,}992{,}225{,}364{,}370{,}600$

$x^{54}y^{24}$

trideca