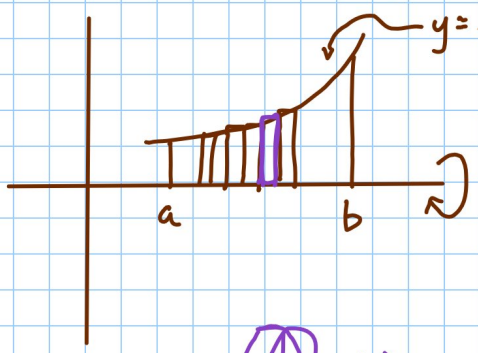
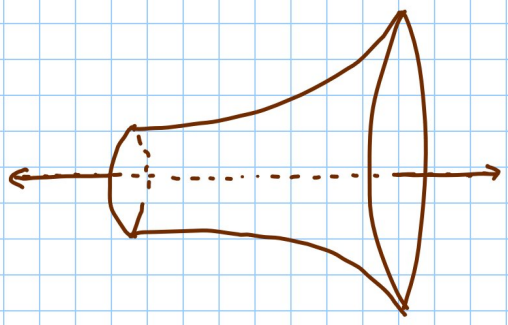
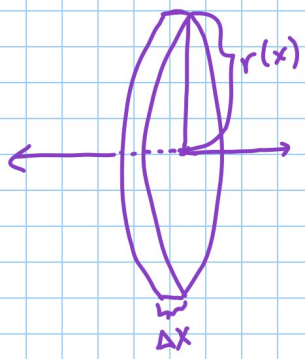
Volume Using Discs / Washers



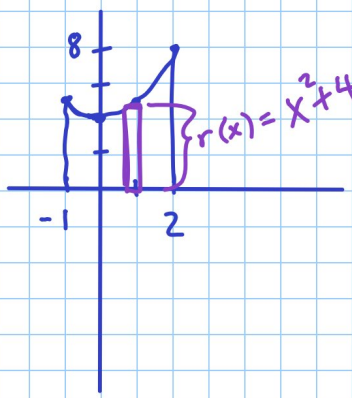


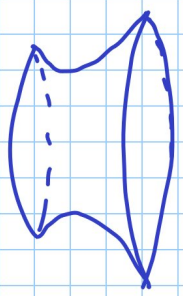


$$V = \pi [r(x)]^2 \Delta x$$

Discs: $V = \pi \int_a^b [r(x)]^2 \, dx$

Ex: Sketch the region bounded by $y = x^2 + 4$, $x = -1$, and $x = 2$, and the x-axis. Find the volume if that region is rotated about the x-axis.

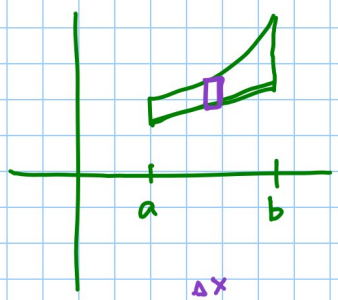


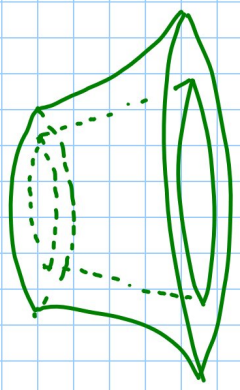


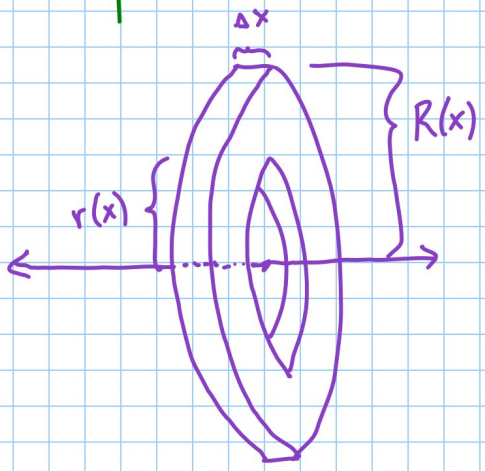
$$V = \pi \int_{-1}^{2} (x^2 + 4)^2 \, dx = \pi \cdot \frac{393}{5} = \frac{393\pi}{5}$$

fnInt $((x^2+4)^2, X, -1, 2)$ ▶FRAC

WASHER







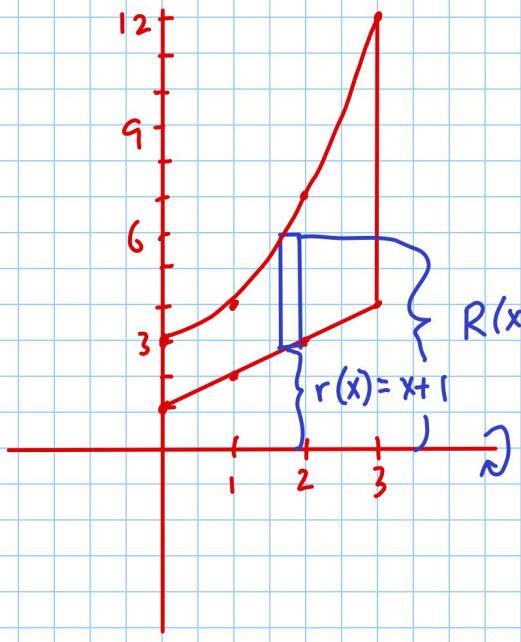
$$\pi\left[R(x)\right]^2\Delta x - \pi\left[r(x)\right]^2\Delta x$$

WASHERS: $V = \pi\int_a^b \left[(R(x))^2 - (r(x))^2\right]dx$

$(\text{outer radius})^2 - (\text{inner radius})^2$

Sketch the region bounded by $y=x^2+3$, $y=x+1$, $x=0$, $x=3$

Find the volume when rotating about the x-axis.

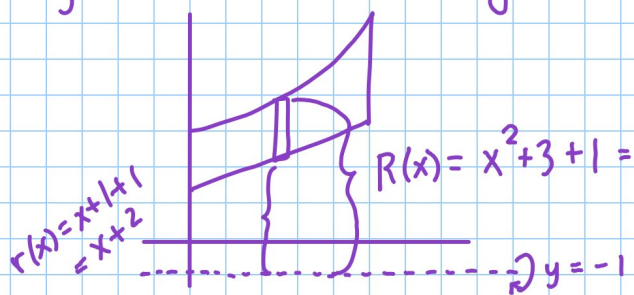


$$V = \pi\int_0^3 \left[(x^2+3)^2 - (x+1)^2\right]dx$$

$$= \frac{543\pi}{5}$$

fnInt $\left((x^2+3)^2-(x+1)^2, X, 0, 3\right)$ ▶FRAC

Same region about the line $y=-1$



$$V = \pi\int_0^3 \left[(x^2+4)^2 - (x+2)^2\right]dx$$

$$= \frac{648\pi}{5}$$

Assignment

Sketch region bounded by $y = 6 - x^2$, $x = -2$, $x = 1$ and the x-axis

A) Find the area

B) Find the volume if region is rotated about

i) x-axis

ii) the line $y = -2$

iii) the line $y = 7$