

1) PARTS:  $\int x^3 \sin(5x) dx$

6)  $\int x^5 \ln x dx$

2) TABLES:  $\int e^{7x} \cos(3x) dx$

7)  $\int x^3 e^{7x} dx$

3) SUBSTITUTION:  $\int x^8 e^{(10-3x^9)} dx$

8)  $\int x^5 \cos(8+3x^6) dx$

4) Partial Fractions  $\int \frac{5x+2}{(x-10)(x+2)} dx$

9)  $\int \frac{2x-3}{x^2-3x-10} dx$

5) TRIG SUB:  $\int \frac{x}{\sqrt{49-x^2}} dx$

10)  $\int x \sqrt{x^2+100} dx$

1)  $\int x^3 \sin(5x) dx = x^3 \left(-\frac{1}{5} \cos(5x)\right) + 3x^2 \cdot \frac{1}{25} \sin(5x) + 6x \cdot \frac{1}{125} \cos(5x) - 6 \cdot \frac{1}{625} \sin(5x) + C$

Sign	u	dv
+	$x^3$	$\sin(5x)$
-	$3x^2$	$-\frac{1}{5} \cos(5x)$
+	$6x$	$-\frac{1}{25} \sin(5x)$
-	$6$	$\frac{1}{125} \cos(5x)$
+	$0$	$\frac{1}{625} \sin(5x)$

$= -\frac{1}{5} x^3 \cos(5x) + \frac{3}{25} x^2 \sin(5x) + \frac{6}{125} x \cos(5x) - \frac{6}{625} \sin(5x) + C$

2)  $\int e^{7x} \cos(3x) dx = \frac{1}{58} e^{7x} [7 \cos(3x) + 3 \sin(3x)] + C$

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$a=7 \quad b=3$

$a^2+b^2=49+9=58$

$$3) \int x^8 e^{(10-3x^9)} dx = -\frac{1}{27} \int e^u du$$

$u = 10 - 3x^9$

$$du = -27x^8 dx$$

$$-\frac{1}{27} du = x^8 dx$$

$$= -\frac{1}{27} e^u + C$$

$$= -\frac{1}{27} e^{(10-3x^9)} + C$$

$$4) \int \frac{5x+2}{(x-10)(x+2)} dx = \int \left( \frac{A}{x-10} + \frac{B}{x+2} \right) dx$$

$$A(x+2) + B(x-10) = 5x+2$$

$$Ax + 2A + Bx - 10B = 5x + 2$$

$$Ax + Bx = 5x$$

$$x(A+B) = 5x$$

$$-2(A+B) = 5$$

$$2A - 10B = 2$$

$$-2A - 2B = -10$$

$$2A - 10B = 2$$

$$-12B = -8$$

$$B = \frac{-8}{-12} = \frac{2}{3}$$

$$A + \frac{2}{3} = 5 \Rightarrow A = \frac{15}{3} - \frac{2}{3}$$

$$A = \frac{13}{3}$$

$$\int \left( \frac{\frac{13}{3}}{x-10} + \frac{\frac{2}{3}}{x+2} \right) dx = \frac{13}{3} \ln|x-10| + \frac{2}{3} \ln|x+2| + C$$

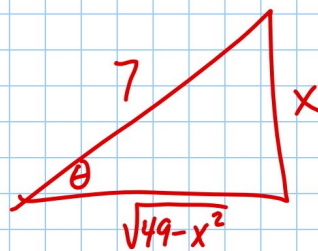
$$5) \int \frac{x}{\sqrt{49-x^2}} dx$$

$$\sin \theta = \frac{x}{7}$$

$$x = 7 \sin \theta$$

$$dx = 7 \cos \theta d\theta$$

$$\sqrt{49-x^2} = 7 \cos \theta$$



$$= \int \frac{7 \sin \theta}{7 \cos \theta} \cdot 7 \cos \theta d\theta$$

$$= \int 7 \sin \theta d\theta$$

$$= -7 \cos \theta + C = -7 \cdot \frac{\sqrt{49-x^2}}{7} + C = -\sqrt{49-x^2} + C$$

$$6) \int x^5 \ln x \, dx = \frac{1}{6} x^6 \ln x - \frac{1}{36} x^6 + C$$

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$$\int x^n \ln x \, dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C$$

$$7) \int x^3 e^{7x} \, dx = \frac{1}{7} x^3 e^{7x} - \frac{3}{49} x^2 e^{7x} + \frac{6}{343} x e^{7x} - \frac{6}{2401} e^{7x} + C$$

sign	u	dv
+	$x^3$	$e^{7x}$
-	$3x^2$	$\frac{1}{7} e^{7x}$
+	$6x$	$\frac{1}{49} e^{7x}$
-	$6$	$\frac{1}{343} e^{7x}$
+	$0$	$\frac{1}{2401} e^{7x}$

$$8) \int x^5 \cos(8+3x^6) \, dx = \frac{1}{18} \int \cos u \, du = \frac{1}{18} \sin u + C$$

$$u = 8 + 3x^6$$

$$du = 18x^5 \, dx$$

$$\frac{1}{18} du = x^5 \, dx$$

$$= \frac{1}{18} \sin(8+3x^6) + C$$

$$9) \int \frac{2x-3}{x^2-3x-10} \, dx = \int \frac{1}{x^2-3x-10} \cdot (2x-3) \, dx = \int \frac{1}{u} \, du$$

$$u = x^2 - 3x - 10$$

$$du = (2x-3) \, dx$$

$$= \ln|u| + C$$

$$= \ln|x^2 - 3x - 10| + C$$

$$10) \int x \sqrt{x^2+100} dx = \int (x^2+100)^{1/2} \cdot \underline{x dx} = \frac{1}{2} \int u^{1/2} du$$

$$u = x^2 + 100$$

$$du = 2x \underline{dx}$$

$$\frac{1}{2} du = x dx$$

$$= \frac{1}{2} \cdot \frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C$$

$$= \frac{1}{3} \left( \sqrt{x^2+100} \right)^3 + C$$

ex:  $\int \frac{1}{x^2-5x-24} dx$