$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

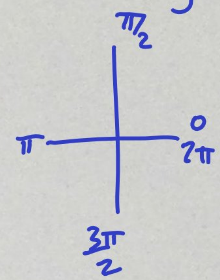
Precalculus B Midterm 3 Practice Test

Name: Key

Complete each of the following problems. Show all necessary work.

In 1-4, use the following information:

$\tan\alpha = \frac{8}{15} = \frac{y}{x}$ with $\underbrace{\pi < \alpha < \frac{3\pi}{2}}_{Q\,III}$ $\sec\beta = \frac{41}{40} = \frac{r}{x}$ with $\overbrace{\frac{3\pi}{2} < \beta < 2\pi}^{Q\,IV}$



$x = -15$ $\sin\alpha = \frac{-8}{17}$

$y = -8$ $\cos\alpha = -\frac{15}{17}$

$r = 17$

$x = 40$ $\sin\beta = -\frac{9}{41}$

$y = -9$

$r = 41$ $\cos\beta = \frac{40}{41}$

1. Find the exact value of $\sin(\alpha + \beta)$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= -\frac{8}{17}\cdot\frac{40}{41} + \left(-\frac{15}{17}\right)\left(-\frac{9}{41}\right) = \frac{-320}{697} + \frac{135}{697} = \boxed{\frac{-185}{697}} = \frac{y}{r}$

2. Find the exact value of $\cos(\alpha + \beta)$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{-15}{17}\cdot\frac{40}{41} - \left(-\frac{8}{17}\right)\left(-\frac{9}{41}\right) = -\frac{600}{697} - \frac{72}{697} = \boxed{\frac{-672}{697}} = \frac{x}{r}$

3. Find the exact value of $\tan(\alpha + \beta)$

$\frac{y}{x} = \frac{-185}{-672} = \boxed{\frac{185}{672}}$

4. In which quadrant is the angle $\alpha + \beta$?

Q III $(x<0, y<0)$

PPP $\sec\alpha = -\frac{17}{8}$ $\pi < \alpha < \frac{3\pi}{2}$

$\cot\beta = \frac{5}{12}$ $0 < \beta < \frac{\pi}{2}$

$\sin(\alpha-\beta)$

$\cos(\alpha-\beta)$

$\tan(\alpha-\beta)$

Q for $\alpha-\beta$

PPP half-angle 5) $\cos 105°$

6) $\sin 15°$

In 5 and 6 use the half-angle formulas to calculate the exact value of each of the following:

5. $\sin 112.5°$

$$\underbrace{\sin 112.5°}_{\substack{Q\,II \\ \sin\theta > 0}} = \sin\frac{225°}{2} = +\sqrt{\frac{1-\cos 225°}{2}}$$

$$= \sqrt{\frac{1-(-\sqrt{2}/2)}{2}} = \sqrt{\frac{(1+\sqrt{2}/2)}{2}\cdot\frac{2}{2}}$$

$$= \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

6. $\cos 165°$

$$\underbrace{}_{\substack{Q\,II \\ \cos\theta < 0}} = \cos\frac{330°}{2} = -\sqrt{\frac{1+\cos 330°}{2}}$$

$$= -\sqrt{\frac{(1+\sqrt{3}/2)}{2}\cdot\frac{2}{2}}$$

$$= -\sqrt{\frac{2+\sqrt{3}}{4}} = -\frac{\sqrt{2+\sqrt{3}}}{2}$$

In 7-10, use the following information: $\cot\theta = \frac{24}{7} \Rightarrow \frac{x}{y}$ ~~with~~ $0 < \theta < \frac{\pi}{2}$

$x = 24$ $\quad \sin\theta = \frac{7}{25}$

$y = 7$

$r = 25$ $\quad \cos\theta = \frac{24}{25}$

7. Find the exact value of $\sin(2\theta)$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$= \frac{2}{1}\left(\frac{7}{25}\right)\left(\frac{24}{25}\right) = \frac{336}{625} = \frac{y}{r}$$

$$(\cos\theta)^2 = \cos^2\theta$$

8. Find the exact value of $\cos(2\theta)$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= \left(\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2 = \frac{576}{625} - \frac{49}{625} = \frac{527}{625} = \frac{x}{r}$$

9. Find the exact value of $\tan(2\theta)$

$$\frac{336}{527}$$

10. Find the exact value of $\tan\frac{\theta}{2}$ $= \frac{1-\cos\theta}{\sin\theta} = \frac{\left(1-\frac{24}{25}\right)25}{\frac{7}{25}\cdot 25} = \frac{25-24}{7} = \frac{1}{7}$

PPP

$\csc\theta = \frac{41}{9}$ $\quad 0 < \theta < \frac{\pi}{2}$

$\sin(2\theta)$, $\cos(2\theta)$, $\tan(2\theta)$, $\tan\frac{\theta}{2}$

In 11 and 12, establish each identity:

11. $\cos^2\theta(1+\tan^2\theta)=1$

$$\cos^2\theta(1+\tan^2\theta) = \cos^2\theta\cdot\sec^2\theta$$
$$= \cos^2\theta\cdot\frac{1}{\cos^2\theta}$$
$$= 1$$

PPP

$$\sin^2\theta(1+\cot^2\theta) = 1$$

12. $\dfrac{\sin^2\theta}{1-\cos\theta}-1=\cos\theta$

$$1-\frac{\cos^2\theta}{1+\sin\theta} = \sin\theta$$

$$\frac{\sin^2\theta}{1-\cos\theta}-1 = \frac{1-\cos^2\theta}{1-\cos\theta}-1$$
$$= \frac{\cancel{(1-\cos\theta)}(1+\cos\theta)}{\cancel{1-\cos\theta}}-1$$
$$= 1+\cos\theta-1$$
$$= \cos\theta$$

In 13 and 14, solve each equation on the interval $[0,2\pi)$.

13. $4\cos^2\theta=3$

$$\cos^2\theta = \frac{3}{4}$$
$$\cos\theta = \pm\sqrt{\frac{3}{4}}$$
$$\cos\theta = \frac{\pm\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

14. $2\sin^2\theta-3\sin\theta+1=0$

$$(2\sin\theta-1)(\sin\theta-1)=0$$

$$2x^2-3x+1$$
$$(2x-1)(x-1)$$

-2 2 -1 -3

$$2x^2-2x-x+1$$
$$2x(x-1)-1(x-1)$$
$$(x-1)(2x-1)$$

$2\sin\theta-1=0$ $\quad$ $\sin\theta-1=0$

$\sin\theta=\frac{1}{2}$ $\quad$ $\sin\theta=1$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$$

PPP $\quad 4\sin^2\theta=3$

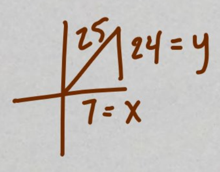
$$2\cos^2\theta-\cos\theta-1=0$$

In 15-20, find the exact value of each:

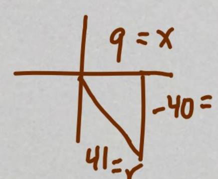
15. $\sin^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$

16. $\sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

17. $\cot\left(\sin^{-1}\frac{\overset{y}{24}}{\underset{r}{25}}\right) = \frac{7}{24}$



18. $\sec\left(\sin^{-1}\left(-\frac{\overset{y}{40}}{\underset{r}{41}}\right)\right) = \frac{41}{9}$



19. $\csc^{-1}(2) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

20. $\cot^{-1}(1) = \tan^{-1}(1) = \frac{\pi}{4}$

15) $\tan^{-1}\sqrt{3}$

16) $\sin^{-1}\left(\sin\frac{11\pi}{6}\right)$

17) $\sec\left(\sin^{-1}\frac{8}{17}\right)$

18) $\cot\left(\tan^{-1}\left(-\frac{24}{7}\right)\right)$

19) $\sec^{-1}(2)$

20) $\cot^{-1}(\sqrt{3})$