SOLUTIONS FOR #1

1) $\int \frac{2x+3}{x^2+3x+2}dx = \int \frac{2x+3}{(x+1)(x+2)}dx = \int \left(\frac{A}{x+1} + \frac{B}{x+2}\right)dx$

$Ax + 2A + Bx + B = 2x + 3$

$A + B = 2 \Rightarrow -A - B = -2$

$2A + B = 3 \qquad 2A + B = 3$

$A = 1$

$1 + B = 2$

$B = 1$

$\int \left(\frac{1}{x+1} + \frac{1}{x+2}\right)dx = \boxed{\ln|x+1| + \ln|x+2| + C}$

2) $\int x^5 \ln x \, dx$

$u = \ln x \quad dv = x^5 dx$

$du = \frac{1}{x}dx \quad v = \frac{x^6}{6}$

$\ln x \cdot \frac{x^6}{6} - \int \frac{x^6}{6} \cdot \frac{1}{x} dx$

$\frac{x^6 \ln x}{6} - \frac{1}{6}\int x^5 dx$

$\boxed{\frac{x^6 \ln x}{6} - \frac{x^6}{36} + C}$

3) $\int \sin^6\theta \cos\theta \, d\theta$

$u = \sin\theta$

$du = \cos\theta \, d\theta$

$\int u^6 du$

$\frac{u^7}{7} + C$

$\boxed{\frac{\sin^7\theta}{7} + C}$

4) $\int \sin(6\theta)\cos\theta \, d\theta$

$$= \frac{1}{1^2 - 6^2} \Rightarrow \boxed{-\frac{1}{35}\left[\sin(6\theta)\sin\theta + 6\cos(6\theta)\cos\theta\right] + C}$$

5) $\int x^4 e^{7x} \, dx$

sign	u	dv
+	x^4	e^{7x}
−	$4x^3$	$\frac{1}{7}e^{7x}$
+	$12x^2$	$\frac{1}{49}e^{7x}$
−	$24x$	$\frac{1}{343}e^{7x}$
+	24	$\frac{1}{2401}e^{7x}$
−	0	$\frac{1}{16807}e^{7x}$

$$= \frac{1}{7}x^4e^{7x} - \frac{4}{49}x^3e^{7x} + \frac{12}{343}x^2e^{7x} - \frac{24}{2401}xe^{7x} + \frac{24}{16807}e^{7x} + C$$

6) $\int \frac{1}{\sqrt{36-x^2}} \, dx$

$x = 6\sin\theta$

$dx = 6\cos\theta \, d\theta$

$\sqrt{36-x^2} = 6\cos\theta$

$\int \frac{1}{\cancel{6\cos\theta}} \cancel{6\cos\theta} \, d\theta$

$\int d\theta$

$= \theta + C$

$\frac{x}{6} = \sin\theta$

$\sin^{-1}\left(\frac{x}{6}\right) = \theta$

$$\boxed{= \sin^{-1}\left(\frac{x}{6}\right) + C}$$

7) $\int \frac{(\ln x)^7}{x}\,dx = \int u^7\,du = \frac{u^8}{8} + C$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\boxed{= \frac{(\ln x)^8}{8} + C}$$

8) $\int x\sqrt{x^2+81}\,dx = \int (x^2+81)^{1/2} \cdot x\,dx$

$u = x^2 + 81$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$= \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2} \cdot \frac{u^{3/2}}{3/2} + C$

$= \frac{1}{2} \cdot \frac{2}{3}\, u^{3/2} + C$

$$\boxed{= \frac{1}{3}\sqrt{(x^2+81)^3} + C}$$

9) $\int \frac{x^3 - 7x^2 + 2}{\sqrt{x}}\,dx = \int \left(x^{5/2} - 7x^{3/2} + 2x^{-1/2}\right)dx$

$= \frac{x^{7/2}}{7/2} - \frac{7x^{5/2}}{5/2} + \frac{2x^{1/2}}{1/2} + C$

$$\boxed{= \frac{2}{7}\sqrt{x^7} - \frac{14}{5}\sqrt{x^5} + 4\sqrt{x} + C}$$

10) $\int x(x^2-5)^2\,dx = \frac{1}{2}\int u^2\,du$

$u = x^2 - 5$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$= \frac{1}{2} \cdot \frac{u^3}{3} + C$

$$\boxed{= \frac{(x^2-5)^3}{6} + C}$$

SOLUTIONS FOR #2

1) $\int x^2\sqrt{7-3x^3}\,dx \quad = -\frac{1}{9}\int u^{1/2}\,du$

$u = 7-3x^3$

$du = -9x^2\,dx$

$-\frac{1}{9}du = x^2\,dx$

$= -\frac{1}{9}\cdot\frac{u^{3/2}}{3/2}+C$

$\boxed{= -\frac{2}{27}\sqrt{(7-3x^3)^3}+C}$

2) $\int x^3 e^{7-x^4}\,dx \quad = -\frac{1}{4}\int e^u\,du$

$u = 7-x^4$

$du = -4x^3\,dx$

$-\frac{1}{4}du = x^3\,dx$

$= -\frac{1}{4}e^u + C$

$\boxed{= -\frac{1}{4}e^{7-x^4}+C}$

3) $\int \cos x\cdot\cos(\sin x)\,dx \quad = \int \cos u\,du$

$u = \sin x$

$du = \cos x\,dx$

$= \sin u + C$

$\boxed{= \sin(\sin x)+C}$

4) $\underbrace{\int \sin^3 x\,dx}_{\#17} = -\frac{1}{3}\sin^2 x\cos x + \frac{2}{3}\int \sin x\,dx$

$\boxed{= -\frac{1}{3}\sin^2 x\cos x - \frac{2}{3}\cos x + C}$

5) $\underbrace{\int e^{5x}\cos(3x)\,dx}_{\#9} = \boxed{\frac{1}{34}e^{5x}\left[5\cos(3x)+3\sin(3x)\right]+C}$

$5^2+3^2 = 34$

6) $\int x^3 e^{5x}\, dx$

sign	u	dv
+	x^3	e^{5x}
−	$3x^2$	$\frac{1}{5}e^{5x}$
+	$6x$	$\frac{1}{25}e^{5x}$
−	6	$\frac{1}{125}e^{5x}$
+	0	$\frac{1}{625}e^{5x}$

$$\boxed{= \frac{1}{5}x^3e^{5x} - \frac{3}{25}x^2e^{5x} + \frac{6}{125}xe^{5x} - \frac{6}{625}e^{5x} + C}$$

7) $\int x^8 \ln x\, dx$

$u = \ln x \qquad dv = x^8\, dx$

$du = \frac{1}{x}\, dx \qquad v = \frac{x^9}{9}$

$$= \ln x \cdot \frac{x^9}{9} - \int \frac{x^9}{9} \cdot \frac{1}{x}\, dx$$

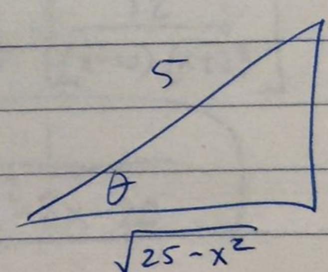
$$= \frac{x^9 \ln x}{9} - \frac{1}{9}\int x^8\, dx$$

$$\boxed{= \frac{x^9 \ln x}{9} - \frac{1}{81}x^9 + C}$$

8) $\int \frac{1}{x^2\sqrt{25-x^2}}\, dx \qquad x = 5\sin\theta$

$dx = 5\cos\theta\, d\theta$

$\sqrt{25-x^2} = 5\cos\theta$



$$= \int \frac{1}{25\sin^2\theta \cdot \cancel{5\cos\theta}} \cdot \cancel{5\cos\theta}\, d\theta$$

$$= \frac{1}{25}\int \csc^2\theta\, d\theta$$

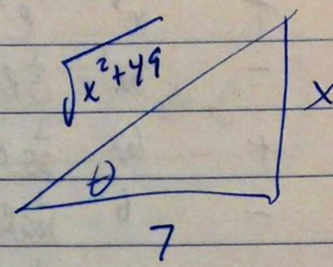
$$= \frac{1}{25}(-\cot\theta) + C$$

$$= -\frac{1}{25} \cdot \frac{\sqrt{25-x^2}}{x} = \boxed{-\frac{\sqrt{25-x^2}}{25x} + C}$$

9) $\int \frac{1}{\sqrt{x^2+49}}\,dx$ $\quad x = 7\tan\theta$

$dx = 7\sec^2\theta\, d\theta$

$\sqrt{x^2+49} = 7\sec\theta$



$= \int \frac{1}{\cancel{7}\sec\theta} \cdot \cancel{7}\sec^2\theta\, d\theta$

$= \int \sec\theta\, d\theta$

$= \ln\left|\sec\theta + \tan\theta\right| + C$

$\boxed{= \ln\left|\frac{\sqrt{x^2+49} + x}{7}\right| + C}$

SOLUTIONS FOR #3

1) $\int \frac{1}{x^2-8x-20}\,dx = \int \frac{1}{(x-10)(x+2)}\,dx = \int\left(\frac{A}{x-10} + \frac{B}{x+2}\right)dx$

$$Ax+2A + Bx - 10B = 1$$

$$-2(A+B=0) \qquad -2A-2B=0$$

$$2A-10B=1 \qquad 2A-10B=1$$

$$-12B = 1$$

$$B = -\frac{1}{12}$$

$$A - \frac{1}{12} = 0$$

$$A = \frac{1}{12}$$

$$\int\left(\frac{\frac{1}{12}}{x-10} + \frac{-\frac{1}{12}}{x+2}\right)dx = \boxed{\frac{1}{12}\ln|x-10| - \frac{1}{12}\ln|x+2| + C}$$

$$\frac{d}{dx}\left[\frac{1}{12}\ln|x-10| - \frac{1}{12}\ln|x+2| + C\right]$$

$$\frac{1}{12}\cdot\frac{1}{x-10} - \frac{1}{12}\cdot\frac{1}{x+2} + 0 = \frac{1}{12}\left[\frac{1}{x-10} - \frac{1}{x+2}\right]$$

$$= \frac{1}{12}\left[\frac{\cancel{x}+2 - \cancel{x}+10}{(x-10)(x+2)}\right]$$

$$= \frac{1}{12}\left[\frac{12}{(x-10)(x+2)}\right]$$

$$\boxed{= \frac{1}{x^2-8x-20}}$$

2) $\int x^2 \sin x \, dx = \boxed{-x^2\cos x + 2x\sin x + 2\cos x + C}$

sign	u	dv
+	x^2	$\sin x$
−	$2x$	$-\cos x$
+	2	$-\sin x$
−	0	$\cos x$

$\frac{d}{dx}\left[-x^2\cos x + 2x\sin x + 2\cos x + C\right]$

$-x^2(-\sin x) + \cancel{\cos x(-2x)} + \cancel{2x\cos x} + \cancel{\sin x\, 2} \cancel{-2\sin x} + \cancel{0}$

$\boxed{= x^2 \sin x}$

3) $\underbrace{\int \sin(5x)\sin(3x)\,dx}_{\#10} = -\frac{1}{16}\left[5\cos(5x)\sin(3x) - 3\sin(5x)\cos(3x)\right] + C$

$b^2 - a^2 = 3^2 - 5^2 = -16$

4) $\int x^2(x^3+5)^{20}\,dx = \frac{1}{3}\int u^{20}\,du$

$u = x^3 + 5$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$= \frac{1}{3}\frac{u^{21}}{21} + C$

$= \frac{(x^3+5)^{21}}{63} + C$

$\frac{d}{dx}\left[\frac{(x^3+5)^{21}}{63} + C\right]$

$\frac{\cancel{21}(x^3+5)^{20} \cdot \cancel{3}x^2}{\cancel{63}} + 0$

$\boxed{x^2(x^3+5)^{20} + C}$

5) $\int \frac{1}{(\sqrt{4-x^2})^3}\, dx$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$\sqrt{4-x^2} = 2\cos\theta$

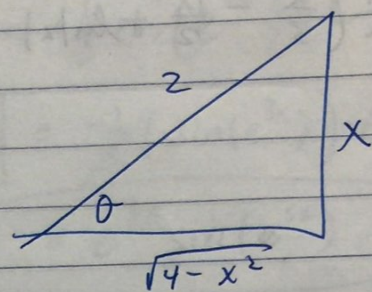
$= \int \frac{1}{8\cos^3\theta} \cdot 2\cos\theta\, d\theta$

$= \frac{1}{4}\int \frac{1}{\cos^2\theta}\, d\theta$

$= \frac{1}{4}\int \sec^2\theta \; \cancel{d\theta} + C$

$= \frac{1}{4}\tan\theta + C$

$$\boxed{= \frac{x}{4\sqrt{4-x^2}} + C}$$



SOLUTIONS FOR #4 (Checks on 1-3)

1) $$\int \frac{x^5-7x^4+x^2}{x^3}\,dx = \int\left(x^2-7x+\frac{1}{x}\right)dx$$

$$\boxed{=\frac{x^3}{3}-\frac{7x^2}{2}+\ln|x|+C}$$

$$\frac{d}{dx}\left(\frac{x^3}{3}-\frac{7x^2}{2}+\ln|x|+C\right)=\frac{3x^2}{3}-\frac{14x}{2}+\frac{1}{x}$$

$$=\left(x^2-7x+\frac{1}{x}\right)\frac{x^3}{x^3}$$

$$=\frac{x^5}{x^3}-\frac{7x^4}{x^3}+\frac{x^2}{x^3}$$

$$\boxed{=\frac{x^5-7x^4+x^2}{x^3}}$$

2) $$\int x(x^2+5)^2\,dx = \frac{1}{2}\int u^2\,du = \frac{1}{2}\cdot\frac{u^3}{3}+C$$

$u=x^2+5$

$du=2x\,dx$

$\frac{1}{2}du= x\,dx$

$$\boxed{=\frac{(x^2+5)^3}{6}+C}$$

$$\frac{d}{dx}\left[\frac{(x^2+5)^3}{6}+C\right]=\frac{\cancel{3}(x^2+5)^2\cdot\cancel{2}x}{\cancel{6}}+0$$

$$\boxed{=x(x^2+5)^2}$$

3) $\int e^{3x}\sin(e^{3x})\,dx = \frac{1}{3}\int \sin u\,du$

$u = e^{3x}$

$du = 3e^{3x}\,dx$

$\frac{1}{3}du = e^{3x}\,dx$

$= -\frac{1}{3}\cos u + C$

$\boxed{= -\frac{1}{3}\cos(e^{3x}) + C}$

$\frac{d}{dx}\left[-\frac{1}{3}\cos(e^{3x}) + C\right] = \frac{-1}{\cancel{3}}(-\sin(e^{3x}))\cdot e^{3x}\cdot\cancel{3}$

$\boxed{= e^{3x}\sin(e^{3x})}$

4) $\int \frac{5x+3}{x^2+7x+12}\,dx = \int \frac{5x+3}{(x+3)(x+4)}\,dx = \int\left(\frac{A}{x+3} + \frac{B}{x+4}\right)dx$

$Ax + 4A + Bx + 3B = 5x + 3$

$-3(A + B = 5)$ $\quad -3A - 3B = -15$

$4A + 3B = 3$ $\quad 4A + 3B = 3$

$A = -12$

$-12 + B = 5$

$B = 17$

$\boxed{-12\ln|x+3| + 17\ln|x+4| + C}$

5) $\int x^3 e^{9x}\,dx = \frac{1}{9}x^3e^{9x} - \frac{1}{27}x^2e^{9x} + \frac{2}{243}xe^{9x} - \frac{2}{2187}e^{9x} + C$

sign	u	dv
+	x^3	e^{9x}
−	$3x^2$	$\frac{1}{9}e^{9x}$
+	$6x$	$\frac{1}{81}e^{9x}$
−	6	$\frac{1}{729}e^{9x}$
+	0	$\frac{1}{6561}e^{9x}$

6) $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx = \int \frac{9\sin^2\theta}{\cancel{3\cos\theta}} \cdot \cancel{3\cos\theta}\,d\theta$

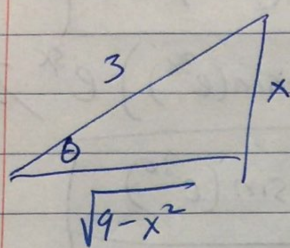
$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$\sqrt{9-x^2} = 3\cos\theta$

$= 9\displaystyle\int \sin^2\theta\,d\theta$

$= 9\left(-\frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\displaystyle\int d\theta\right) + C$



$= -\frac{9}{2}\sin\theta\cos\theta + \frac{9}{2}\theta + C$

$= \frac{-\cancel{9}}{2} \cdot \frac{x}{\cancel{3}} \cdot \frac{\sqrt{9-x^2}}{\cancel{3}} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + C$

$$\boxed{= -\frac{x\sqrt{9-x^2}}{2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + C}$$

SOLUTIONS TO #5

1) $\int \cos^5 x \, dx = \frac{1}{5}\cos^4 x \sin x + \frac{4}{5}\int \cos^3 x \, dx$

$= \frac{1}{5}\cos^4 x \sin x + \frac{4}{5}\left(\frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\int \cos x \, dx\right)$

$\boxed{= \frac{1}{5}\cos^4 x \sin x + \frac{4}{15}\cos^2 x \sin x + \frac{8}{15}\sin x + C}$

2) $\int \frac{x}{\sqrt{x^2+81}} \, dx = \int \frac{x}{(x^2+81)^{1/2}} \, dx = \int x(x^2+81)^{-1/2} \, dx$

$u = x^2 + 81$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$= \frac{1}{2}\int u^{-1/2} \, du$

$= \cancel{\frac{1}{2}} \cdot \frac{u^{1/2}}{\cancel{1/2}} + C$

$\boxed{= \sqrt{x^2+81} + C}$

3) $\int \left(6\cos x + \frac{5}{x} + 3\sqrt[5]{x^7}\right) dx$

$\cancel{3} \cdot \frac{5}{\cancel{12}_{4}} = \frac{5}{4}$

$\int \left(6\cos x + \frac{5}{x} + 3x^{7/5}\right) + C$

$6\sin x + 5\ln x + \frac{3x^{12/5}}{12/5} + C$

$\boxed{6\sin x + 5\ln|x| + \frac{5}{4}\sqrt[5]{x^{12}} + C}$

4) $\underbrace{\int x^6 e^{2x}\,dx}_{\#14}$

$\frac{1}{2}x^6e^{2x} - \frac{1}{4}\cdot 6x^5e^{2x} + \frac{1}{8}\cdot 30x^4e^{2x} - \frac{1}{16}\cdot 120x^3e^{2x} + \frac{1}{32}\cdot 360x^2e^{2x}$

$- \frac{1}{64}\cdot 720xe^{2x} + \frac{1}{128}\cdot 720e^{2x} + C$

$$\boxed{= \frac{1}{2}x^6e^{2x} - \frac{3}{2}x^5e^{2x} + \frac{15}{4}x^4e^{2x} - \frac{15}{2}x^3e^{2x} + \frac{45}{4}x^2e^{2x} - \frac{45}{4}xe^{2x} + \frac{45}{8}e^{2x} + C}$$

5) $\int \overbrace{x^3 \ln x}^{\#13}\,dx$

$$\frac{x^4\ln x}{4} - \frac{x^4}{16} + C$$

SOLUTIONS TO #6

$\int \cos^7\theta \sin\theta \, d\theta = -\int u^7 du$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$= -\frac{u^8}{8} = \boxed{-\frac{\cos^8\theta}{8} + C}$

$\frac{d}{d\theta}\left[-\frac{\cos^8\theta}{8} + C\right] = -\frac{\cancel{8}\cos^7\theta}{\cancel{8}} \cdot (-\sin\theta)$

$\boxed{= \cos^7\theta \sin\theta}$

$\underbrace{\int \sin(7\theta)\cos(4\theta)\,d\theta}_{\#12} = -\frac{1}{33}\left[4\sin(7\theta)\sin(4\theta) + 7\cos(7\theta)\cos(4\theta)\right] + C$

$b^2 - a^2 = 4^2 - 7^2 = 16 - 49$

$= -33$

$\underbrace{\int e^{7x}\cos(4x)\,dx}_{\#9} = \frac{1}{65}e^{7x}\left[7\cos(4x) + 4\sin(4x)\right] + C$

$a^2 + b^2 = 7^2 + 4^2 = 65$

4) $\displaystyle\int \frac{x^3}{\sqrt[4]{8-x^4}}\,dx = -\frac{1}{4}\int u^{-1/4}\,du$

$u = 8-x^4$

$du = -4x^3\,dx$

$-\frac{1}{4}\,du = x^3\,dx$

$= -\frac{1}{4}\dfrac{u^{3/4}}{3/4} + C$

$= -\frac{1}{4}\cdot\frac{4}{3}\left(\sqrt[4]{8-x^4}\right)^3 + C$

$$\boxed{= -\frac{1}{3}\left(\sqrt[4]{8-x^4}\right)^3 + C}$$

Check: $\dfrac{d}{dx}\left[-\frac{1}{3}(8-x^4)^{3/4} + C\right]$

$= \cancel{-\frac{1}{3}}\,\cancel{\frac{3}{4}}\,(8-x^4)^{-1/4}\cdot(\cancel{-4}x^3)$

$$\boxed{= \frac{x^3}{\sqrt[4]{8-x^4}}}$$

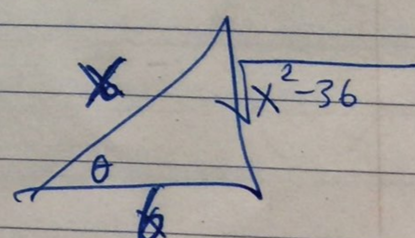
5) $\displaystyle\int \frac{x}{\sqrt{x^2-36}}\,dx$

$x = 6\sec\theta$

$dx = 6\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-36} = 6\tan\theta$

$\displaystyle\int \frac{6\sec\theta}{\cancel{6\tan\theta}}\cdot\cancel{6}\sec\theta\,\cancel{\tan\theta}\,d\theta$



$= 6\displaystyle\int \sec^2\theta\,d\theta$

$= 6\tan\theta + C$

$= \dfrac{6\sqrt{x^2-36}}{6} + C$

$$\boxed{= \sqrt{x^2-36} + C}$$

SOLUTIONS TO #7

1) $a(t) = -32$

$v(t) = -32t + 80$

$h(t) = -16t^2 + 80t + 96$

a) $v(t) = 0$

$-32t + 80 = 0$

$t = \frac{80}{32} = \boxed{2.5\,\text{sec}}$

b) $h(2.5) = -16(2.5)^2 + 80(2.5) + 96$

$\boxed{= 196\text{ ft}}$

c) $-16t^2 + 80t + 96 = 0$

$-16(t^2 - 5t - 6) = 0$

$-16(t-6)(t+1) = 0$

$\boxed{t = 6 \text{ seconds}}$

d) $v(6) = -32 \cdot 6 + 80$

$\boxed{= -112 \text{ ft/sec}}$

2) $\int \frac{5}{x^2+6x+8}\,dx = \int \left(\frac{A}{x+4} + \frac{B}{x+2}\right) dx$

$Ax + 2A + Bx + 4B = 5$

$-2(A + B = 0) \rightarrow \cancel{-2A} - 2B = 0$

$2A + 4B = 5 \qquad \cancel{2A} + 4B = 5$

$2B = 5$

$B = 5/2$

$A + 5/2 = 0$

$A = -5/2$

$\int \left(\frac{-5/2}{x+4} + \frac{5/2}{x+2}\right) dx = \boxed{\frac{-5}{2}\ln|x+4| + \frac{5}{2}\ln|x+2| + C}$

3) $\int (5x^2+3x)e^{4x}\,dx = \boxed{\frac{1}{4}(5x^2+3x)e^{4x} - \frac{1}{16}(10x+3)e^{4x} + \frac{5}{32}e^{4x} + C}$

sign	u	dv
+	$5x^2+3x$	e^{4x}
−	$10x+3$	$\frac{1}{4}e^{4x}$
+	10	$\frac{1}{16}e^{4x}$
−	0	$\frac{1}{64}e^{4x}$

4) $\displaystyle\int \frac{x}{\sqrt{49+x^2}}\,dx$ $\quad x = 7\tan\theta$

$dx = 7\sec^2\theta\,d\theta$

$\sqrt{49+x^2} = 7\sec\theta \Rightarrow \sec\theta = \dfrac{\sqrt{49+x^2}}{7}$

$\displaystyle\int \frac{7\tan\theta}{\cancel{7}\cancel{\sec\theta}} \cdot \cancel{7}\sec^{\cancel{2}}\theta\,d\theta$

$= 7\displaystyle\int \sec\theta\tan\theta\,d\theta$

$= 7\sec\theta + C$

$= 7 \cdot \dfrac{\sqrt{49+x^2}}{7} + C = \boxed{\sqrt{49+x^2} + C}$

5) $\displaystyle\int \frac{\cos x}{\sqrt{\sin x}}\,dx = \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du$

$u = \sin x$

$du = \cos x\,dx$

$= \dfrac{u^{1/2}}{1/2} + C$

$= 2\sqrt{u} + C$

$\boxed{= 2\sqrt{\sin x} + C}$

Solutions to #8

1) $\int \cos^{10}\theta \sin\theta \, d\theta = -\int u^{10} \, du$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$= -\dfrac{u^{11}}{11} + C$

$$\boxed{= \frac{-\cos^{11}\theta}{11} + C}$$

2) $\int \cos(10\theta)\sin\theta \, d\theta = \underbrace{\int \sin\theta \cos(10\theta) \, d\theta}_{\#12}$

$b^2 - a^2 = 10^2 - 1^2 = 99$

$$\boxed{= \frac{1}{99}\left[10\sin\theta\sin(10\theta) + \cos\theta\cos(10\theta)\right] + C}$$

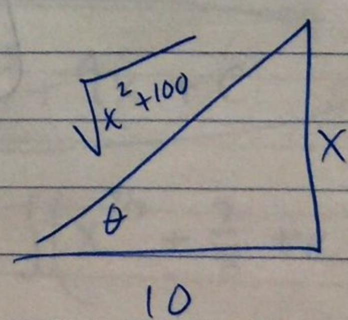
3) $\int \dfrac{1}{(\sqrt{x^2+100})^3} \, dx$

$x = 10\tan\theta$

$dx = 10\sec^2\theta \, d\theta$

$\sqrt{x^2+100} = 10\sec\theta$

$= \int \dfrac{1}{1000\sec^3\theta} \cdot 10\sec^2\theta \, d\theta$



$= \dfrac{1}{100}\int \dfrac{1}{\sec\theta} \, d\theta$

$= \dfrac{1}{100}\int \cos\theta \, d\theta$

$= \dfrac{1}{100}\sin\theta + C = \dfrac{1}{100} \cdot \dfrac{x}{\sqrt{x^2+100}}$ $\boxed{= \dfrac{x}{100\sqrt{x^2+100}} + C}$

4) $\int x^3 e^{5x}\,dx$

sign	u	dv
$+$	x^3	e^{5x}
$-$	$3x^2$	$\frac{1}{5}e^{5x}$
$+$	$6x$	$\frac{1}{25}e^{5x}$
$-$	6	$\frac{1}{125}e^{5x}$
$+$	0	$\frac{1}{625}e^{5x}$

$$= \frac{1}{5}x^3e^{5x} - \frac{3}{25}x^2e^{5x} + \frac{6}{125}xe^{5x} - \frac{6}{625}e^{5x} + C$$

5) $\int x^5\sqrt{x^6-5}\,dx = \int (x^6-5)^{1/2}\cdot x^5\,dx$

$u = x^6 - 5$

$du = 6x^5\,dx$

$$= \frac{1}{6}\int u^{1/2}\,du$$

$$= \frac{1}{6}\cdot\frac{u^{3/2}}{3/2} + C$$

$$= \frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{6}_{3}}u^{3/2} + C$$

$$\boxed{= \frac{1}{9}\left(\sqrt{x^6-5}\right)^3 + C}$$

6) $\int \frac{3x+2}{(x-5)(x+6)}\,dx = \int\left(\frac{A}{x-5} + \frac{B}{x+6}\right)dx$

$Ax+6A + Bx - 5B = 3x+2$

$-5(A+B=3) \Rightarrow +5A+5B = +15$

$6A-5B=2$ $\qquad$ $6A-5B=2$

$11A = 17$

$A = \frac{17}{11}$

$\frac{17}{11} + B = 3$

$B = \frac{33}{11} - \frac{17}{11} = \frac{16}{11}$

$\int\left(\frac{\frac{17}{11}}{x-5} + \frac{\frac{16}{11}}{x+6}\right)dx$

$$\boxed{\frac{17}{11}\ln|x-5| + \frac{16}{11}\ln|x+6| + C}$$

7) $\int (x^2-3)^2\,dx = \int(x^4-6x^2+9)\,dx$

$= \frac{x^5}{5} - \frac{6x^3}{3} + 9x + C$

$= \frac{x^5}{5} - 2x^3 + 9x + C$

8) $\int\left(\sqrt[3]{x^4} + \frac{7}{x} + e^{3x}\right)dx = \int\left(x^{4/3} + \frac{7}{x} + e^{3x}\right)dx$

$= \frac{x^{7/3}}{7/3} + 7\ln|x| + \frac{1}{3}e^{3x} + C$

$$\boxed{= \frac{3}{7}\sqrt[3]{x^7} + 7\ln|x| + \frac{1}{3}e^{3x} + C}$$