WARMUP

1) $\int x^3 e^{2x}\,dx$

19) $\int \frac{\ln x}{x^2}\,dx = \int x^{-2}\ln x\,dx = \ln x\left(-\frac{1}{x}\right) - \int\left(-\frac{1}{x}\cdot\frac{1}{x}\right)dx$

$u = \ln x$ $\quad dv = x^{-2}\,dx$

$du = \frac{1}{x}\,dx$ $\quad V = \frac{x^{-1}}{-1} = -\frac{1}{x}$

$$\frac{-\ln x}{x} + \int x^{-2}\,dx$$

$$= -\frac{\ln x}{x} + \frac{x^{-1}}{-1} + C$$

$$= \frac{-\ln x}{x} - \frac{1}{x} + C$$

$$= \frac{-\ln x - 1}{x} + C$$

Section 7.3 Table of Integrals

ex: $\int \sin(7x)\sin(3x)\,dx$ $\quad b^2 - a^2 = 3^2 - 7^2 = 9 - 49 = -40$

$a = 7 \quad b = 3$

#10) $\int \sin(ax)\sin(bx)\,dx = \frac{1}{b^2 - a^2}\left[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)\right] + C$

$$= -\frac{1}{40}\left[7\cos(7x)\sin(3x) - 3\sin(7x)\cos(3x)\right] + C$$

ex: $\displaystyle\int (x^2-3x+2)e^{3x}\,dx$ $\quad a=3$

$$\#14)\quad \int p(x)e^{ax}\,dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \dots$$

$$= \frac{1}{3}(x^2-3x+2)e^{3x} - \frac{1}{9}(2x-3)e^{3x} + \frac{1}{27}2e^{3x} + C$$

$$= \frac{1}{3}(x^2-3x+2)e^{3x} - \frac{1}{9}(2x-3)e^{3x} + \frac{2}{27}e^{3x} + C$$

$n=6$

ex: $\displaystyle\int \sin^6 x\,dx$

$$\#17)\quad \int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$$

$$= -\frac{1}{6}\sin^5 x\cos x + \frac{5}{6}\int \sin^4 x\,dx$$

$$= -\frac{1}{6}\sin^5 x\cos x + \frac{5}{6}\left(-\frac{1}{4}\sin^3 x\cos x + \frac{3}{4}\int \sin^2 x\,dx\right)$$

$$= -\frac{1}{6}\sin^5 x\cos x - \frac{5}{24}\sin^3 x\cos x + \frac{5}{8}\int \sin^2 x\,dx$$

$\sin^0 x$

$$= -\frac{1}{6}\sin^5 x\cos x - \frac{5}{24}\sin^3 x\cos x + \frac{5}{8}\left(-\frac{1}{2}\sin x\cos x + \frac{1}{2}\int 1\,dx\right)$$

$$\boxed{\int \sin^6 x\,dx = -\frac{1}{6}\sin^5 x\cos x - \frac{5}{24}\sin^3 x\cos x - \frac{5}{16}\sin x\cos x + \frac{5}{16}x + C}$$

Check: $\displaystyle\frac{d}{dx}\left[-\frac{1}{6}\sin^5 x\cos x - \frac{5}{24}\sin^3 x\cos x - \frac{5}{16}\sin x\cos x + \frac{5}{16}x + C\right]$

$$= -\frac{1}{6}\sin^5 x(-\sin x) + \cos x\left(-\frac{5}{6}\sin^4 x\cdot\cos x\right) - \frac{5}{24}\sin^3 x(-\sin x) + \cos x\left(-\frac{5}{8}\sin^2 x\cdot\cos x\right)$$
$$- \frac{5}{16}\sin x(-\sin x) + \cos x\left(-\frac{5}{16}\cos x\right) + \frac{5}{16} + 0$$

$$= \frac{1}{6}\sin^6 x - \frac{5}{6}\sin^4 x\cos^2 x + \frac{5}{24}\sin^4 x - \frac{5}{8}\sin^2 x\cos^2 x + \frac{5}{16}\sin^2 x - \frac{5}{16}\cos^2 x + \frac{5}{16}$$

$$= \frac{1}{6}\sin^6 x - \frac{5}{6}\sin^4 x(1-\sin^2 x) + \frac{5}{24}\sin^4 x - \frac{5}{8}\sin^2 x(1-\sin^2 x) + \frac{5}{16}\sin^2 x - \frac{5}{16}(1-\sin^2 x) + \frac{5}{16}$$

$$= \frac{1}{6}\sin^6 x - \frac{5}{6}\sin^4 x + \frac{5}{6}\sin^6 x + \frac{5}{24}\sin^4 x - \frac{5}{8}\sin^2 x + \frac{5}{8}\sin^4 x + \frac{5}{16}\sin^2 x - \cancel{\frac{5}{16}} + \frac{5}{16}\sin^2 x + \cancel{\frac{5}{16}}$$

$$= \left(\frac{1}{6} + \frac{5}{6}\right)\sin^6 x + \cancel{\left(-\frac{5}{6} + \frac{5}{24} + \frac{5}{8}\right)\sin^4 x} + \cancel{\left(-\frac{5}{8} + \frac{5}{16} + \frac{5}{16}\right)\sin^2 x}$$

$$\cancel{-\frac{20}{24} + \frac{5}{24} + \frac{15}{24}} \qquad \cancel{-\frac{10}{16} + \frac{5}{16} + \frac{5}{16}}$$

$$= \sin^6 x$$

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$$\int x^3 e^{2x}\,dx = x^3 \cdot \frac{1}{2}e^{2x} - 3x^2 \cdot \frac{1}{4}e^{2x} + 6x \cdot \frac{1}{8}e^{2x} - 6 \cdot \frac{1}{16}e^{2x} + C$$

$$= \frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \frac{3}{8}e^{2x} + C$$

sign		u		dv
$+$	$\rightarrow$	x^3	$\searrow$	e^{2x}
$-$	$\rightarrow$	$3x^2$	$\searrow$	$\frac{1}{2}e^{2x}$
$+$	$\rightarrow$	$6x$	$\searrow$	$\frac{1}{4}e^{2x}$
$-$	$\rightarrow$	6	$\searrow$	$\frac{1}{8}e^{2x}$
$+$		0		$\frac{1}{16}e^{2x}$