WARMUP

So far we have learned to calculate antiderivatives by basic formulas and by substitution.
Which method would you choose for the following?
If neither apply, write neither.

1) $\int \frac{x^2}{\sqrt{5-x^3}}\,dx$

2) $\int \sin^3 x \cos x\,dx$

3) $\int \frac{(x^4-3)^2}{x}\,dx$

4) $\int xe^x\,dx$

5) $\int xe^{x^2}\,dx$

6) $\int x^3 \ln x\,dx$

Section 7.2 Integration By Parts

$$\int u\,dv = uv - \int v\,du$$

ex: $\int xe^x\,dx$

Choose u and dv

$u = x$ (derivative) $\quad dv = e^x dx$ (antiderivative)

$du = dx \quad v = e^x$

$$\int u\,dv = uv - \int v\,du$$
$$= xe^x - \int e^x\,dx$$
$$= xe^x - e^x + C$$

Check: $\frac{d}{dx}\left[xe^x - e^x + C\right]$

$= xe^x + e^x \cdot 1 - e^x + 0$

$= xe^x$

How to choose u and dv:

- Whatever you let dv be, you need to be able to find v
- It helps if du is simpler than u
- It helps if v is simpler than dv

ex: $\int \theta \cos\theta \, d\theta$

$u = \theta \qquad dv = \cos\theta \, d\theta$

$du = d\theta \qquad v = \sin\theta$

$\theta \sin\theta - \int \sin\theta \, d\theta$

$u \cdot v - \int v \, du$

$\theta \sin\theta + \cos\theta + C$

ex: $\int x^3 \ln x \, dx$

Any time there's an $\ln x$, let $u = \ln x$

$u = \ln x \qquad dv = x^3 \, dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^4}{4}$

$\frac{\ln x}{1} \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$

$\frac{x^4 \ln x}{4} - \frac{1}{4} \int x^3 \, dx$

$\frac{x^4 \ln x}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + C$

$$\boxed{\frac{x^4 \ln x}{4} - \frac{x^4}{16} + C}$$

ex: $\int x^4 \sin x\,dx = x^4(-\cos x) - \int(-\cos x)\cdot 4x^3\,dx$

$u = x^4$ $\quad dv = \sin x\,dx$

$du = 4x^3\,dx$ $\quad v = -\cos x$

$= -x^4\cos x + \int 4x^3\cos x\,dx$

uh-oh gotta do u and dv again

ALTERNATE PARTS PROCESS – TABULAR METHOD

sign	u	dv
+	x^4	$\sin x$
−	$4x^3$	$-\cos x$
+	$12x^2$	$-\sin x$
−	$24x$	$\cos x$
+	24	$\sin x$
−	0	$-\cos x$

$x^4(-\cos x) - 4x^3(-\sin x) + 12x^2\cos x - 24x\sin x + 24(-\cos x) + C$

$-x^4\cos x + 4x^3\sin x + 12x^2\cos x - 24x\sin x - 24\cos x + C$

DON'T USE TABULAR WITH LN PROBLEMS

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$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

3) $\int t^2 e^{5t}\,dt$

sign $\quad$ u $\quad$ dv

+	t^2	e^{5t}
−	$2t$	$\frac{1}{5}e^{5t}$
+	2	$\frac{1}{25}e^{5t}$
−	0	$\frac{1}{125}e^{5t}$