9) $\int y^2(1+y)^2\,dy = \int y^2(1+2y+y^2)\,dy$

$$= \int (y^2+2y^3+y^4)\,dy$$

$$= \frac{y^3}{3} + \frac{2y^4}{4} + \frac{y^5}{5} + C$$

$$= \frac{y^3}{3} + \frac{y^4}{2} + \frac{y^5}{5} + C$$

16) $\int (x^2+3)^2\,dx$

$$= \int (x^4+6x^2+9)\,dx$$

$$= \frac{x^5}{5} + 2x^3 + 9x + C$$

$$\int \frac{(\ln x)^2}{x}\,dx = \int (\ln x)^2 \cdot \frac{1}{x}\,dx$$

$u = \ln x$

$$= \int u^2 \cdot du$$

$du = \frac{1}{x}\,dx$

$$= \frac{u^3}{3} + C$$

$$= \frac{(\ln x)^3}{3} + C$$

When we substitute, we are trying to make the integral look like one of these

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x \cos(e^x)\,dx = \int \cos(e^x)\cdot e^x\,dx$$

$\cancel{u = \cos(e^x)}$

$\cancel{du = -\sin(e^x)\cdot e^x\,dx}$

$$= \int \cos u\,du$$

$u = e^x$

$du = e^x dx$

$= \sin u + C$

$= \sin(e^x) + C$

ex: $\int \sin^6 x \cdot \cos x \, dx = \int (\sin x)^6 \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int u^6 \, du$

$= \frac{u^7}{7} + C$

$= \frac{\sin^7 x}{7} + C$

$u = \sin x$

$du = \cos x \, dx$

$\frac{du}{\cos x} = dx$

$\int u^6 \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}}$

$\int u^6 \, du$

ex: $\int \frac{e^x}{e^x + 5} dx = \int \frac{1}{e^x + 5} \cdot e^x dx$

$u = e^x + 5$

$du = e^x dx$

$= \int \frac{1}{u} du$

$= \ln|u| + C$

$= \ln(e^x + 5) + C$

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19) $\int x e^{-x^2} dx$

$u = -x^2$

$du = -2x \, dx$

$\frac{du}{-2x} = dx$

$\int \cancel{x} e^u \cdot \frac{du}{-2\cancel{x}}$

$-\frac{1}{2} \int e^u du = -\frac{1}{2} e^u = -\frac{1}{2} e^{-x^2} + C$

6) $\int t \cos(t^2) dt = \int \cos(t^2) \cdot t \, dt$

$u = t^2$

$du = 2t \, dt$

$\frac{1}{2} du = t \, dt$

$\frac{1}{2} \int \cos u \cdot du$

$\frac{1}{2} \sin u + C$

$\frac{1}{2} \sin(t^2) + C$

29) $\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy = \int e^{\sqrt{y}} \frac{1}{\sqrt{y}}\,dy$

$u = \sqrt{y} = y^{\frac{1}{2}}$

$du = \frac{1}{2} y^{-\frac{1}{2}}\,dy$

$2 \cdot du = \frac{1}{2} \cdot \frac{1}{\sqrt{y}}\,dy \cdot 2$

$2\,du = \frac{1}{\sqrt{y}}\,dy$

$= \int e^{u} \cdot 2\,du$

$= 2\int e^{u}\,du$

$= 2e^{u} + C$

$= 2e^{\sqrt{y}} + C$

$\frac{4}{3} \cdot 7$

$4 \cdot \frac{1}{3} \cdot 7$