WARMUP

Find a function whose derivative is each of the following:

1) $f'(x) = 3x^2 \Rightarrow f(x) = x^3$

2) $g'(x) = 6x^5 \Rightarrow g(x) = x^6$

3) $h'(x) = x^3 \Rightarrow h(x) = \frac{1}{4}x^4$

4) $g'(x) = \cos x \Rightarrow g(x) = \sin x$

5) $f'(x) = \frac{1}{x} \Rightarrow f(x) = \ln x$

6) $f'(x) = e^x \Rightarrow f(x) = e^x$

23) AVG VALUE on $[1,6] = 4$ $\quad 5 \cdot \overbrace{\frac{1}{6-1}\int_1^6 v(x)\,dx}$ $= 4 \cdot 5 \Rightarrow \int_1^6 v(x)\,dx = 20$

AVG VALUE on $[6,8] = 5$ $\quad 2 \cdot \frac{1}{8-6}\int_6^8 v(x)\,dx = 5 \cdot 2 \Rightarrow \int_6^8 v(x)\,dx = 10$

AVG VALUE on $[1,8] = ?$

so $\int_1^8 v(x)\,dx = 20 + 10 = 30$

$$\frac{(6-1)\cdot 4 + (8-6)\cdot 5}{8-1} = \frac{30}{7}$$

so avg value $= \frac{1}{8-1} \cdot 30 = \frac{30}{7}$

22) $\int_a^a f(x)\,dx = -\int_a^a f(x)\,dx$

$2\int_a^a f(x)\,dx = 0$

$\int_a^a f(x)\,dx = 0$

Section 6.2 Constructing Antiderivatives Analytically

indefinite integral it's asking you to find an antiderivative $\rightarrow$ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$$\int k\,dx = kx + C$$

ex: $\int (3x^2 + 5x + 3)\,dx = \frac{3x^3}{3} + \frac{5x^2}{2} + 3x + C$

$$= x^3 + \frac{5}{2}x^2 + 3x + C$$

ex: $\int \frac{7}{x^2}\,dx = \int 7x^{-2}\,dx = \frac{7x^{-1}}{-1} + C$

$$= -\frac{7}{x} + C$$

ex: $\int 4\sqrt[5]{x^8}\,dx = \int 4x^{8/5}\,dx = \frac{4x^{13/5}}{\frac{13}{5}} + C$ $\quad 13/5 \leftarrow \frac{8}{5}+1$

$$= \frac{5}{13}\cdot 4x^{13/5} + C$$

$$= \frac{20}{13}\sqrt[5]{x^{13}} + C$$

ex: $\int \underbrace{(x^2+5)^2\,dx}_{\text{FOIL}} = \int (x^4 + 10x^2 + 25)\,dx$

$$= \frac{x^5}{5} + \frac{10x^3}{3} + 25x + C$$

OR

$= \frac{1}{5}x^5 + \frac{10}{3}x^3 + 25x + C$

$(x^2+5)(x^2+5) = x^4 + 10x^2 + 25$

ex: $\int \frac{x^7 + 3x^5 - 7x^4}{x^2}\,dx = \int\left(\frac{x^7}{x^2} + \frac{3x^5}{x^2} - \frac{7x^4}{x^2}\right)dx$

$= \int(x^5 + 3x^3 - 7x^2)\,dx$

$= \frac{x^6}{6} + \frac{3x^4}{4} - \frac{7x^3}{3} + C$

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1, 3, 9, 13, 19, 21, 25, 29, 45, 47

1) $f(x) = 5$

$\int 5\,dx = 5x + C$

9) $\int \frac{1}{x^3}\,dx$

$\int x^{-3}\,dx$

$\frac{x^{-2}}{-2} + C$

$\frac{1}{-2x^2} + C$

47) $\int 4\sqrt{w}\,dw = \int 4w^{\frac{1}{2}}\,dw = \frac{4w^{3/2}}{3/2} = \frac{8}{3}\sqrt{w^3} + C$

$x^{\frac{1}{2}}$

$\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$