

## WARMUP

$$1) \int e^x \cos(e^x + 2) dx = \int \cos u du = \sin u + C$$
$$u = e^x + 2 \quad = \sin(e^x + 2) + C$$
$$du = e^x dx$$

$$2) \int \frac{x^2 - 3}{x^3 - 9x + 4} dx = \int \frac{1}{x^3 - 9x + 4} \cdot (x^2 - 3) dx$$
$$u = x^3 - 9x + 4 \quad = \int \frac{1}{u} \cdot \frac{1}{3} du$$
$$du = (3x^2 - 9) dx$$
$$\frac{1}{3} du = (x^2 - 3) dx \quad = \frac{1}{3} \int \frac{1}{u} du$$
$$= \frac{1}{3} \ln |u| + C$$
$$= \frac{1}{3} \ln |x^3 - 9x + 4| + C$$

## Integration by Partial Fractions

ex:  $\int \frac{1}{x^2 - 10x + 9} dx = \int \frac{1}{(x-1)(x-9)} dx$

$$\frac{1}{(x-1)(x-9)} = \frac{A}{x-1} + \frac{B}{x-9}$$

$$\frac{1}{(x-1)(x-9)} = \frac{A(x-9) + B(x-1)}{(x-1)(x-9)}$$

$$A(x-9) + B(x-1) = 1$$

$$Ax - 9A + Bx - B = 1$$

$$Ax + Bx - 9A - B = 1$$

$$\frac{5}{6} \times \frac{1}{7}$$

$$\frac{35 + 6}{6 \cdot 7} = \frac{41}{42}$$

- ① Factor bottom
- ② Write as sum of fractions
- ③ Cross-multiply
- ④ Simplify
- ⑤ Set x-terms equal  
constants equal
- ⑥ Solve for A and B

$$(A+B)x + (-9A-B) = 0x + 1 \quad \textcircled{7} \text{ Anti-differentiate}$$

$$\begin{array}{r} A+B = 0 \\ -9A-B = 1 \\ \hline -8A = 1 \\ A = -\frac{1}{8} \end{array} \quad \begin{array}{l} \text{Add eqs together} \\ -\frac{1}{8} + B = 0 \\ B = \frac{1}{8} \end{array}$$

$$\frac{d}{dx}(\ln|x-1|) = \frac{1}{x-1}$$

$$\int \frac{1}{(x-1)(x-9)} dx = \int \left( \frac{-\frac{1}{8}}{x-1} + \frac{\frac{1}{8}}{x-9} \right) dx$$

$$= -\frac{1}{8} \ln|x-1| + \frac{1}{8} \ln|x-9| + C$$

$$\text{ex: } \int \frac{1}{x^2-2x-8} dx = \int \frac{A}{x-4} + \frac{B}{x+2} dx$$

$$A(x+2) + B(x-4) = 1$$

$$\begin{array}{r} -2(A+B = 0) \\ 2A-4B = 1 \end{array} \quad \begin{array}{r} -2A-2B = 0 \\ 2A-4B = 1 \\ \hline -6B = 1 \\ B = -\frac{1}{6} \end{array} \quad \begin{array}{r} A - \frac{1}{6} = 0 \\ A = \frac{1}{6} \end{array}$$

$$= \int \left( \frac{\frac{1}{6}}{x-4} + \frac{-\frac{1}{6}}{x+2} \right) dx = \frac{1}{6} \ln|x-4| - \frac{1}{6} \ln|x+2| + C$$

$$\text{ex: } \int \frac{4x+1}{x^2+4x-21} dx = \int \frac{4x+1}{(x-3)(x+7)} dx = \int \left( \frac{A}{x-3} + \frac{B}{x+7} \right) dx$$

$$A(x+7) + B(x-3) = 4x+1$$

$$\begin{array}{r} (A+B = 4) \cdot 3 \rightarrow 3A+3B = 12 \\ 7A-3B = 1 \rightarrow 7A-3B = 1 \\ \hline 10A = 13 \end{array}$$

$$A = \frac{13}{10}$$

$$\frac{13}{10} + B = 4$$

$$B = \frac{40}{10} - \frac{13}{10} = \frac{27}{10}$$

$$\int \left( \frac{\frac{13}{10}}{x-3} + \frac{\frac{27}{10}}{x+7} \right) dx = \frac{13}{10} \ln |x-3| + \frac{27}{10} \ln |x+7| + C$$

ASSIGNMENT Use Partial Fractions

$$1) \int \frac{1}{x^2 - 2x - 35} dx$$

$$2) \int \frac{5}{x^2 - 3x - 10} dx$$

$$3) \int \frac{5x + 2}{x^2 + 8x + 7} dx$$