

Section 8.4 (continued and concluded)

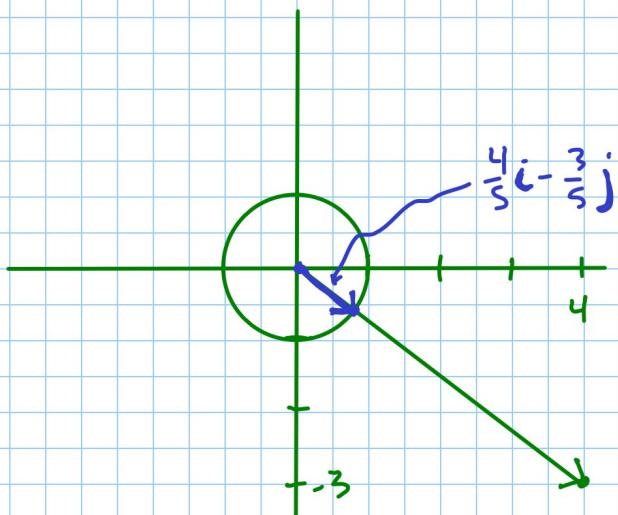
For any nonzero vector \mathbf{v} , the vector $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$ is a unit vector that has the same direction as \mathbf{v} and a magnitude of 1.

ex: Find the unit vector in the same direction as

$$\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$$

$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{4^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$

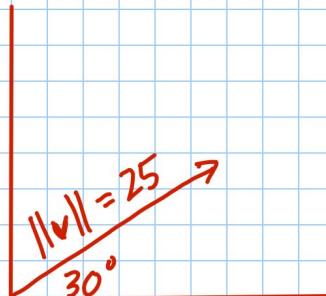
$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{4\mathbf{i} - 3\mathbf{j}}{5} = \frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$$



Often the direction of the vector \mathbf{v} is given by the angle \mathbf{v} makes with the positive x -axis. If this angle is α , then \mathbf{v} can be expressed by

$$\mathbf{v} = \|\mathbf{v}\| (\cos \alpha \mathbf{i} + \sin \alpha \mathbf{j})$$

ex: You throw a ball at 25 mph so the angle is 30° with the positive x -axis.



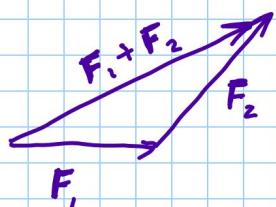
$$\mathbf{v} = 25 (\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j})$$

$$\mathbf{v} = 25 (0.866\mathbf{i} + 0.5\mathbf{j})$$

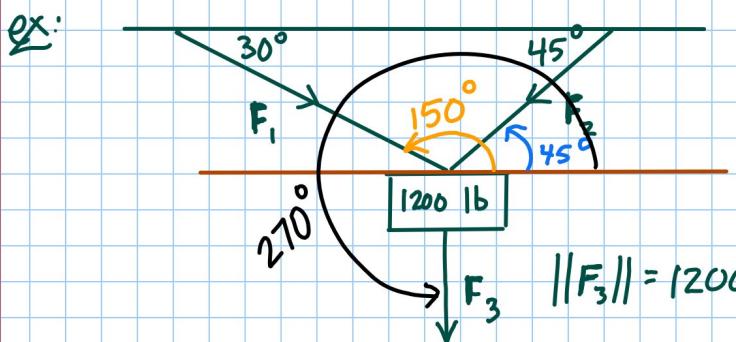
$$\mathbf{v} = 21.65\mathbf{i} + 12.5\mathbf{j}$$

In Physics, force \mathbf{F} is a vector. If 2 different forces \mathbf{F}_1 and \mathbf{F}_2 act on an object, the resultant force is

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$



An object is in equilibrium if
 1) the sum of all forces is $\mathbf{0}$
 2) the object is at rest



$$\mathbf{F}_1 = \|\mathbf{F}_1\| (\cos 150^\circ \mathbf{i} + \sin 150^\circ \mathbf{j}) = -0.866 \|\mathbf{F}_1\| \mathbf{i} + 0.5 \|\mathbf{F}_1\| \mathbf{j}$$

$$\mathbf{F}_2 = \|\mathbf{F}_2\| (\cos 45^\circ \mathbf{i} + \sin 45^\circ \mathbf{j}) = 0.707 \|\mathbf{F}_2\| \mathbf{i} + 0.707 \|\mathbf{F}_2\| \mathbf{j}$$

$$\mathbf{F}_3 = 1200 (\underbrace{\cos 270^\circ \mathbf{i}}_{-1} + \underbrace{\sin 270^\circ \mathbf{j}}_{-1}) = -1200 \mathbf{j}$$

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0} = 0\mathbf{i} + 0\mathbf{j}$$

$$\text{Let } x = \|\mathbf{F}_1\|$$

$$y = \|\mathbf{F}_2\|$$

$$\|\mathbf{F}_1\| = 878.5 \text{ lbs}$$

$$\|\mathbf{F}_2\| = 1076.1 \text{ lbs}$$

$$\begin{aligned} -0.866x + 0.707y &= 0 \\ 0.5x + 0.707y - 1200 &= 0 \end{aligned}$$

$$\begin{aligned} -0.866x + 0.707y &= 0 \\ 0.5x + 0.707y &= 1200 \\ \hline -1.366x &= -1200 \end{aligned}$$

$$x = 878.5$$

$$-.866(878.5) + .707y = 0$$

$$-760.78 + .707y = 0$$

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$$.707y = 760.78$$

$$y = 1076.1$$