Section 8.4 Day 2

$\|\mathbf{v}\|$ represents the magnitude of $\mathbf{v}$.

Properties: a) $\|\mathbf{v}\| \geq 0$

b) $\|\mathbf{v}\| = 0$ if and only if $\mathbf{v} = \mathbf{0}$

c) $\|-\mathbf{v}\| = \|\mathbf{v}\|$

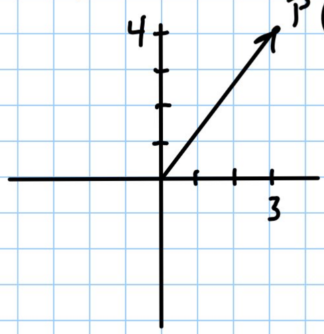
d) $\|\alpha \mathbf{v}\| = |\alpha| \cdot \|\mathbf{v}\|$

A vector $\mathbf{u}$ for which $\|\mathbf{u}\| = 1$ is called a <u>unit vector</u>

We can represent vectors algebraically by breaking them down into <u>components</u>

$\mathbf{v} = \langle a, b \rangle$ a and b are real numbers and are the components of $\mathbf{v}$

We use the coordinate plane to represent algebraic vectors. If $\mathbf{v} = \langle a, b \rangle$ has its initial point at the origin, then $\mathbf{v}$ is a position vector.

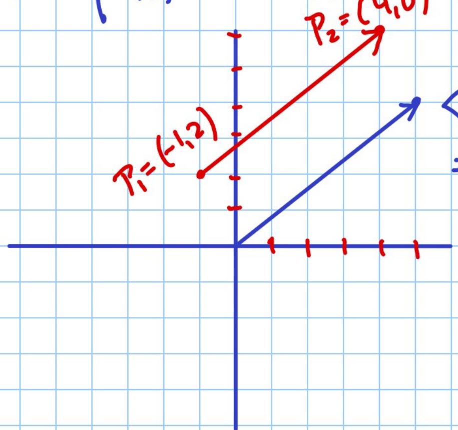


position vector $\langle 3, 4 \rangle$

Any vector whose initial point is not at the origin is equal to a position vector.

Suppose $\mathbf{v}$ is a vector with initial point $P_1 = (x_1, y_1)$ and terminal point $P_2 = (x_2, y_2)$

If $\mathbf{v} = \overrightarrow{P_1P_2}$, then $\mathbf{v}$ is equal to the position vector $\langle x_2 - x_1, y_2 - y_1 \rangle$



$\langle 4-(-1), 6-2 \rangle$
$= \langle 5, 4 \rangle$

We can also write any algebraic vector as $a\mathbf{i} + b\mathbf{j}$ where $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$. $\mathbf{i}$ and $\mathbf{j}$ are unit vectors.

In other words $\mathbf{v} = \langle a, b \rangle = a\mathbf{i} + b\mathbf{j} = a\langle 1, 0 \rangle + b\langle 0, 1 \rangle$

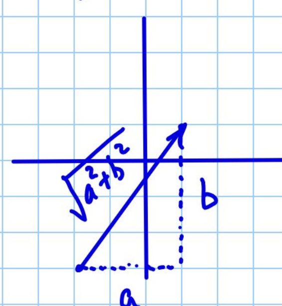
Let $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j} = \langle a_1, b_1 \rangle$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j} = \langle a_2, b_2 \rangle$ and α is a scalar.

$$\mathbf{v} + \mathbf{w} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j} = \langle a_1 + a_2, b_1 + b_2 \rangle$$

$$\mathbf{v} - \mathbf{w} = (a_1 - a_2)\mathbf{i} + (b_1 - b_2)\mathbf{j} = \langle a_1 - a_2, b_1 - b_2 \rangle$$

$$\alpha\mathbf{v} = (\alpha a_1)\mathbf{i} + (\alpha b_1)\mathbf{j} = \langle \alpha a_1, \alpha b_1 \rangle$$

$$\|\mathbf{v}\| = \sqrt{a_1^2 + b_1^2}$$



ex 3 p624 If $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} = \langle 2, 3 \rangle$

$\mathbf{w} = 3\mathbf{i} - 4\mathbf{j} = \langle 3, -4 \rangle$

a) $v + w = \langle 2, 3\rangle + \langle 3, -4\rangle = \langle 5, -1\rangle = 5i - j$

b) $v - w = -i + 7j$

c) $3v = 3\langle 2, 3\rangle = \langle 6, 9\rangle = 6i + 9j$

d) $2v - 3w = 2\langle 2, 3\rangle - 3\langle 3, -4\rangle$

$= \langle 4, 6\rangle - \langle 9, -12\rangle$

$= \langle -5, 18\rangle = -5i + 18j$

e) $\|v\| = \sqrt{2^2 + 3^2} = \sqrt{13}$