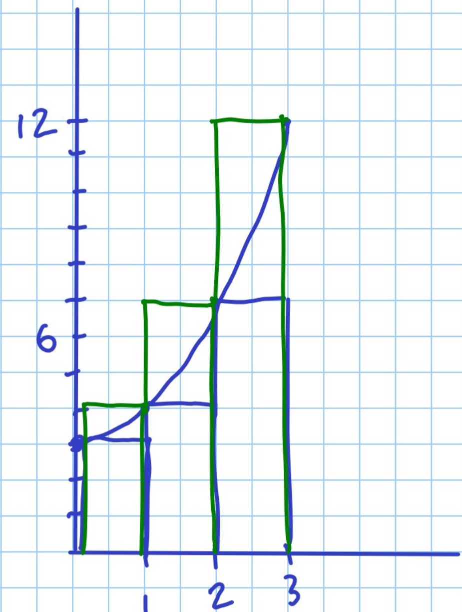
WARMUP

Graph $f(t) = t^2 + 3$ on the interval $[0, 3]$.

Calculate left-hand and right-hand sums using 3 intervals.



left-hand = 14 = 3+4+7

Right-hand = 23 = 4+7+12

$$\int_0^3 (t^2+3)\,dt$$

$$\left[\frac{t^3}{3} + 3t\right]_0^3 = \left(\frac{3^3}{3} + 3\cdot 3\right) - \left(\frac{0^3}{3} + 3\cdot 0\right)$$

$$= 18$$

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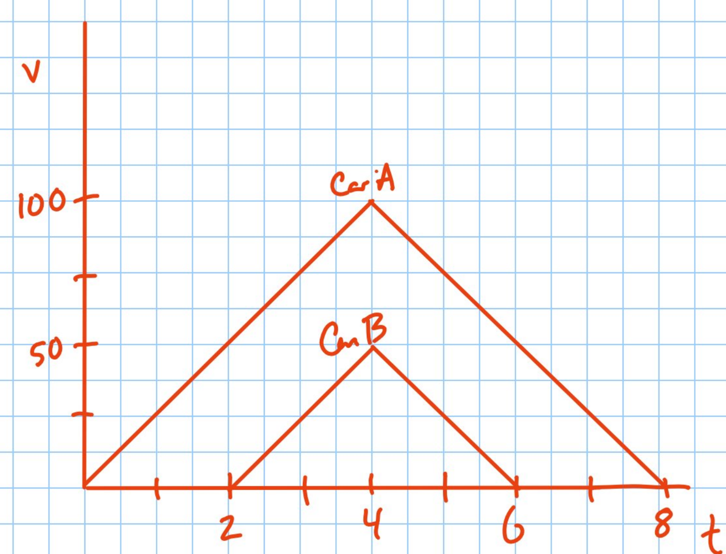
6)



a) Car A 8 hours

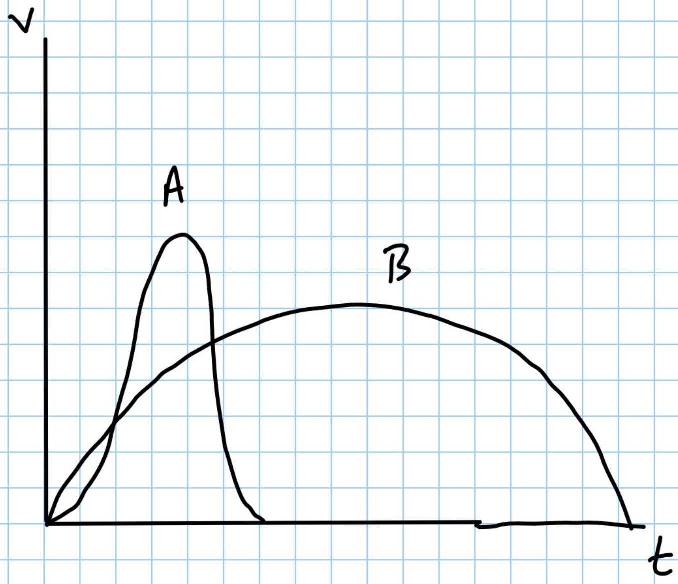
Car B 4 hours

b) 100 km/hr

c) Car B = $\frac{1}{2} \cdot 4 \cdot 50 = 100$ km

Car A = $\frac{1}{2} \cdot 8 \cdot 100 = 400$ km

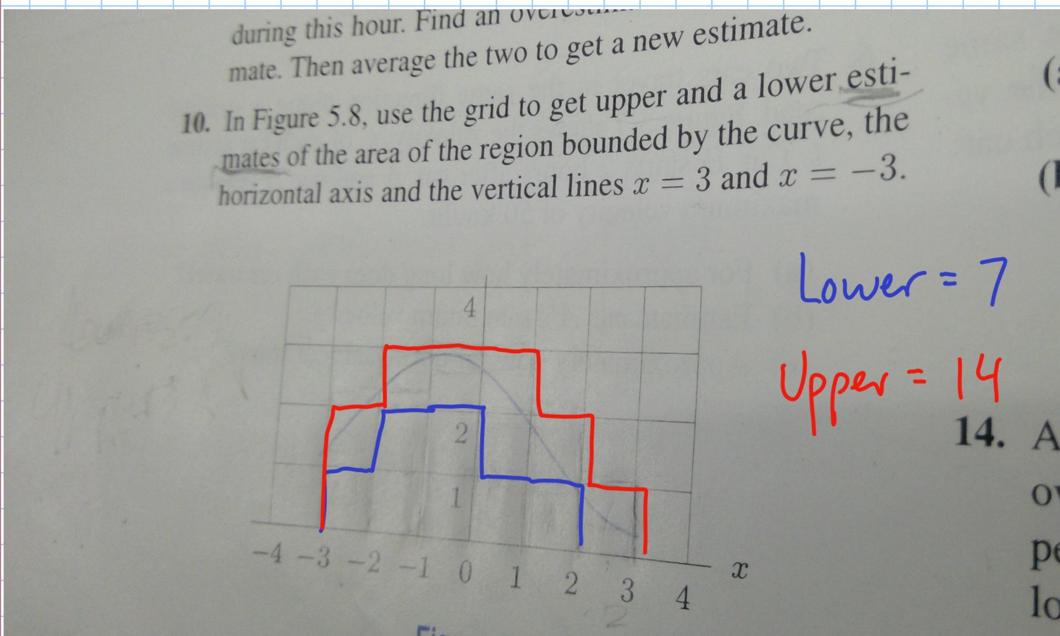
5)



a) A

b) A

c) B

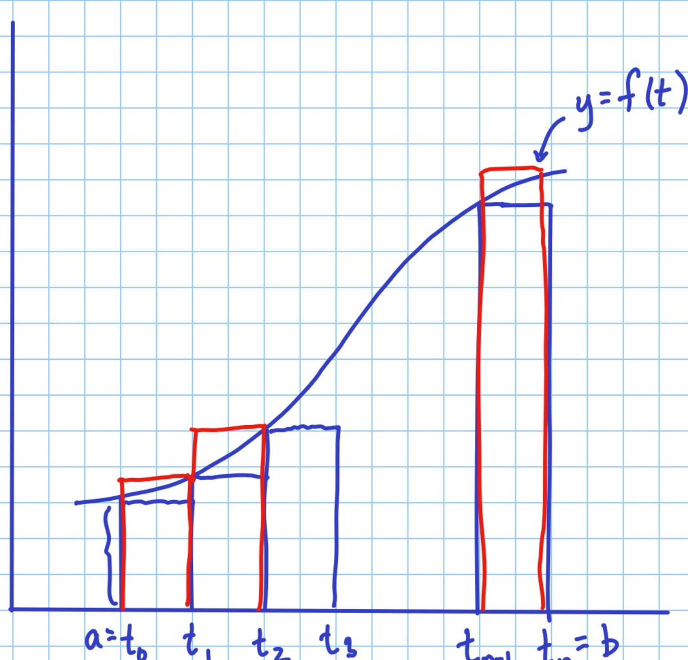

during this hour. Find an overestimate and an underestimate. Then average the two to get a new estimate.

10. In Figure 5.8, use the grid to get upper and a lower estimates of the area of the region bounded by the curve, the horizontal axis and the vertical lines $x = 3$ and $x = -3$.

Lower = 7

Upper = 14

Section 5.2 The Definite Integral



$$\Delta t = t_i - t_{i-1}$$

Left-hand sum

$$= \lim_{n\to\infty} \left[f(t_0)\Delta t + f(t_1)\Delta t + \dots + f(t_{n-1})\Delta t \right]$$

$$= \lim_{n\to\infty} \left[\sum_{i=0}^{n-1} f(t_i)\Delta t \right]$$

Right-hand sum

$$= \lim_{n\to\infty} \left[f(t_1)\Delta t + f(t_2)\Delta t + \dots + f(t_n)\Delta t \right]$$

$= \lim_{n\to\infty} \left[\sum_{i=1}^{n} f(t_i)\Delta t\right]$

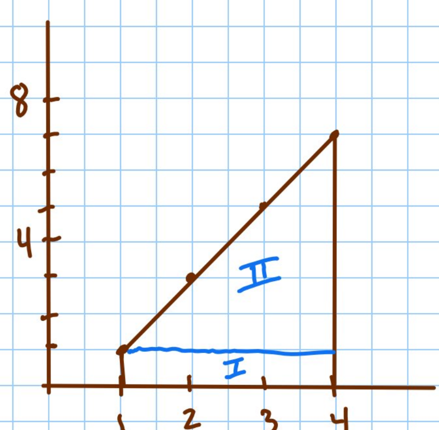
The definite integral is $\int_a^b f(t)\,dt$. f is continuous on $[a,b]$. $\int$ is the integral sign. a is the lower limit of integration and b is the upper limit of integration.

If $f(t) \geq 0$ on $[a,b]$, then $\int_a^b f(t)\,dt$ is the area under the curve from $x=a$ to $x=b$.

$$\text{So } \int_a^b f(t)\,dt = \underbrace{\lim_{n\to\infty}\left[\sum_{i=0}^{n-1} f(t_i)\Delta t\right] = \lim_{n\to\infty}\left[\sum_{i=1}^{n} f(t_i)\Delta t\right]}_{\text{Riemann Sum}}.$$

ex: $\int_1^4 (2x-1)\,dx \Rightarrow$ Area under $f(x)=2x-1$ from $x=1$ to $x=4$

① Step 1: Graph the function on the interval.

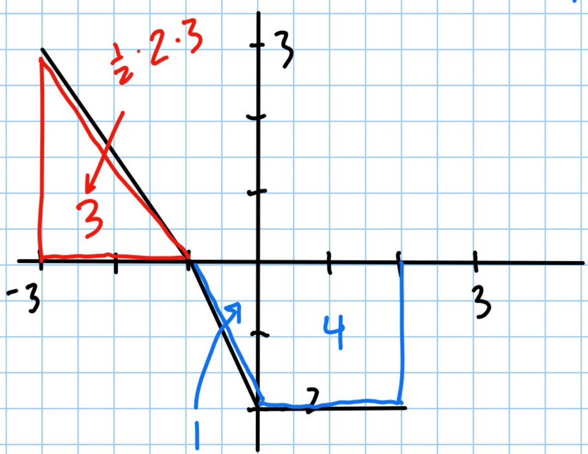


② Step 2: Calculate the area between $f(x)=2x-1$, $x=1$, $x=4$, and the x-axis.

Area of I: $A = 3\cdot 1 = 3$

Area of II: $A = \frac{1}{2}\cdot 3\cdot 6 = 9$

Therefore $\int_1^4 (2x-1)\,dx = 3+9 = 12$



If $f(x) \le 0$ on $[a,b]$ then $\int_a^b f(x)\,dx$ is the negative of the area above the curve.

$$\int_{-1}^{2} f(x)\,dx = -5$$

$$\int_{-3}^{-1} f(x)\,dx = 3$$

so $\int_{-3}^{2} f(x)\,dx = -5+3 = -2$

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2, 3, 7, 30-32