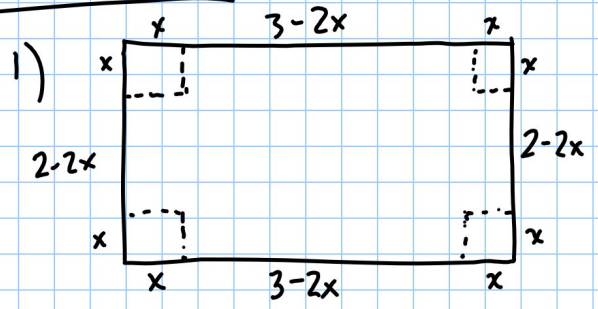
Solution to 21

1) 

2) Constraint: Making box from $3' \times 2'$

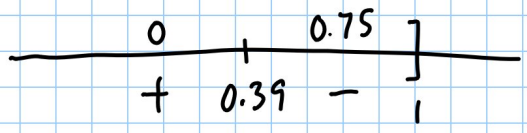
3) $V = x(3-2x)(2-2x)$

4-6) $V = x(6-10x+4x^2)$

$V = 6x - 10x^2 + 4x^3$

$V' = 6 - 20x + 12x^2 = 0$

$x = 0.39, \ 1.27$

7) 

8) $0.39'$

$2-2(0.39) = 1.22'$

$3-2(0.39) = 2.22'$

$0.39' \times 1.22' \times 2.22'$

ex: Find 2 positive numbers whose product is 192 and whose sum is a minimum.

1) no picture

2) constraint: $xy = 192$

3) primary: $S = x + y$

4) $xy = 192$

$x = \frac{192}{y}$

$$S = \frac{192}{y} + y$$

5) $S = 192y^{-1} + y$

$$S' = -192y^{-2} + 1 = 0 \Rightarrow 1 - \frac{192}{y^2}$$

6) $$1 = \frac{192}{y^2}$$

$$y^2 = 192$$

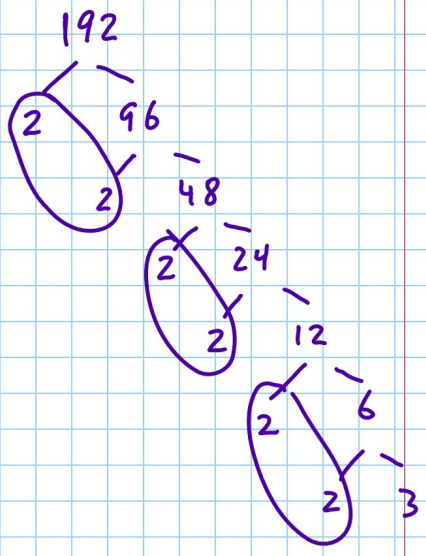
$$y = \sqrt{192} = \sqrt{64}\sqrt{3}$$

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

$$y = \sqrt{4}\sqrt{4}\sqrt{4}\sqrt{3}$$

$$y = 2 \cdot 2 \cdot 2\sqrt{3}$$

$$y = 8\sqrt{3}$$



7) $\frac{1 \qquad 15}{- \quad 8\sqrt{3} \quad +}$

$y = 8\sqrt{3}$ yields a min

8) $$x = \frac{192}{8\sqrt{3}} = \frac{96}{4\sqrt{3}} = \frac{48}{2\sqrt{3}} = \frac{24}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}$$

2 numbers are $8\sqrt{3}$ and $8\sqrt{3}$

p219-220

22, 25, 32

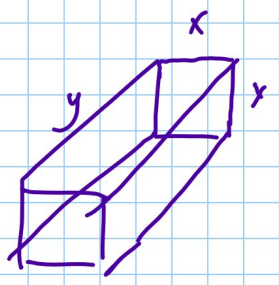
STICKELERS [sic].
by Terry Stickels

Supposedly, the following puzzle was a question on the 1869 MIT admissions test.

The perpendicular dropped from the vertex of the right angle upon the *hypotenuse* divides it into two segments of 9 and 16 respectively. Find the length of the perpendicular (P), and the two legs of the triangle.

A

22)



Constraint: $x^2y = 83\frac{1}{3}$

Primary: $A = x^2 + 3xy$