STEPS for Graphing

STEP 1: Determine intercepts using calculator

STEP 2: Determine asymptotes if the function is a fraction:

VAs: set denominator = 0

H.A.: find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$

STEP 3: Determine extrema

- critters: domain values for which $f'(x)=0$ or $f'(x)$ undefined
- number line: include critters and VAs
- determine where max or min

STEP 4: Determine IPs:

- PIPs: domain values for which $f''(x)=0$ or $f''(x)$ undefined
- number line: include PIPs and asymptotes
- determine IPs

STEP 5: Graph It!

ex: $f(x) = x^3 - 9x^2 - 21x + 189$

STEP 1: x-int(s): $x^3 - 9x^2 - 21x + 189 = 0$

$x^2(x-9) - 21(x-9) = 0$

$(x-9)(x^2-21) = 0$

$x-9=0 \qquad x^2-21=0$

$x=9 \qquad x^2=21$

$x = \pm\sqrt{21} = \pm 4.6$

$(9,0), (4.6,0), (-4.6,0)$

y-int:

$f(0) = 0^3 - 9\cdot0^2 - 21\cdot0 + 189$

$f(0) = 189$

$(0,189)$

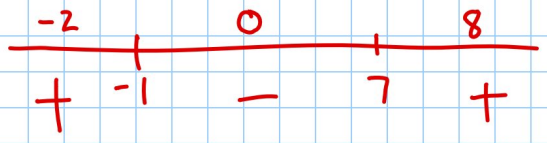
STEP 2: does not apply

STEP 3: $f'(x) = 3x^2 - 18x - 21 = 0$

$3(x^2 - 6x - 7) = 0$

$3(x-7)(x+1) = 0$

critters are 7 and -1



relative max @ $(-1, f(-1)) = (-1, 200)$

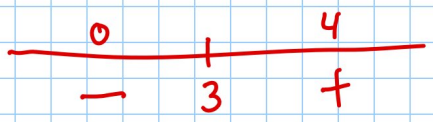
relative min @ $(7, f(7)) = (7, -56)$

STEP 4: IPs

$f''(x) = 6x - 18 = 0$

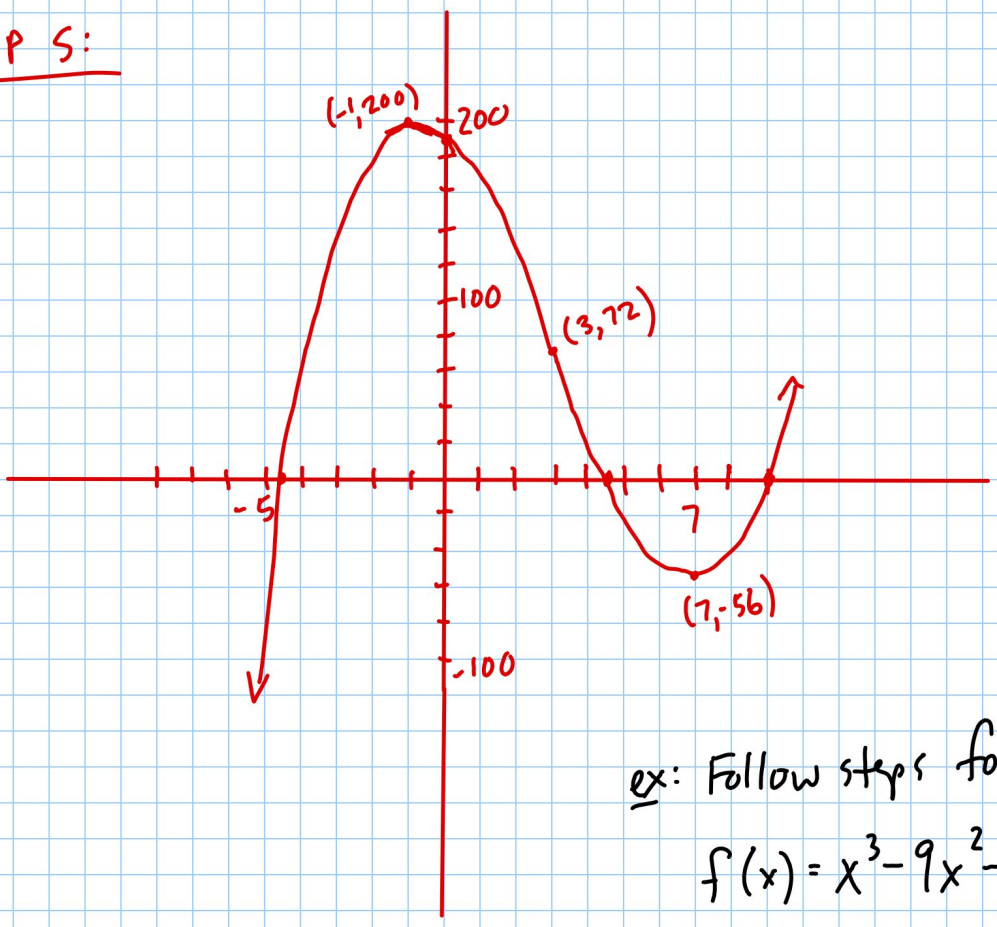
$6x = 18$

$x = 3$



IP @ $(3, f(3)) = (3, 72)$

STEP 5:



ex: Follow steps for

$f(x) = x^3 - 9x^2 - 48x + 52$

ex: Graph $f(x) = \frac{2(x^2-9)}{x^2-4}$

intercepts: x-ints · set top = 0

$2(x^2-9) = 0$

$2(x+3)(x-3) = 0$

$(-3,0)\ (3,0)$

y-int: set $x = 0$

$y = \frac{2(0^2-9)}{0^2-4} = \frac{-18}{-4} = \frac{9}{2}$

$(0, \frac{9}{2})$

Asymptotes: V.A.s - set bottom = 0

$x^2-4 = 0$

$x^2 = 4$

$x = \pm 2$

H.A. $\lim_{x\to\infty} \frac{(2x^2-18)}{(x^2-4)} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$\lim_{x\to\infty} \frac{2-\frac{18}{x^2}}{1-\frac{4}{x^2}} = \frac{2-0}{1-0} = 2$

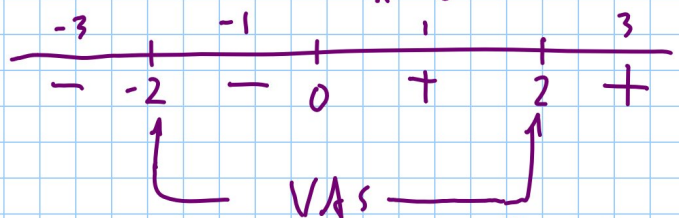
$y = 2$

Extrema: $f'(x) = \frac{(x^2-4)4x - (2x^2-18)2x}{(x^2-4)^2}$

$f'(x) = \frac{4x^3-16x-4x^3+36x}{(x^2-4)^2} = \frac{20x}{(x^2-4)^2}$

$f'(x) = 0$ when $20x = 0$

$x = 0$



graph may change from inc to dec or vice versa around VAs

min @ $(0, f(0)) = (0, 4.5)$

IPs: $f''(x) = \frac{(x^2-4)^2 \cdot 20 - 20x \cdot 2(x^2-4) \cdot 2x}{(x^2-4)^4}$

$f''(x) = \frac{20(x^2-4)^2 - 80x^2(x^2-4)}{(x^2-4)^4}$

$$f''(x) = \frac{20(x^2-4)\left[x^2-4-4x^2\right]}{(x^2-4)^{4\cdot 3}} = \frac{20(-3x^2-4)}{(x^2-4)^3}$$

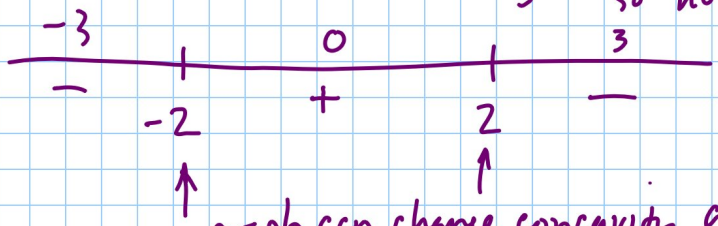
set top = 0

$$-3x^2-4=0$$

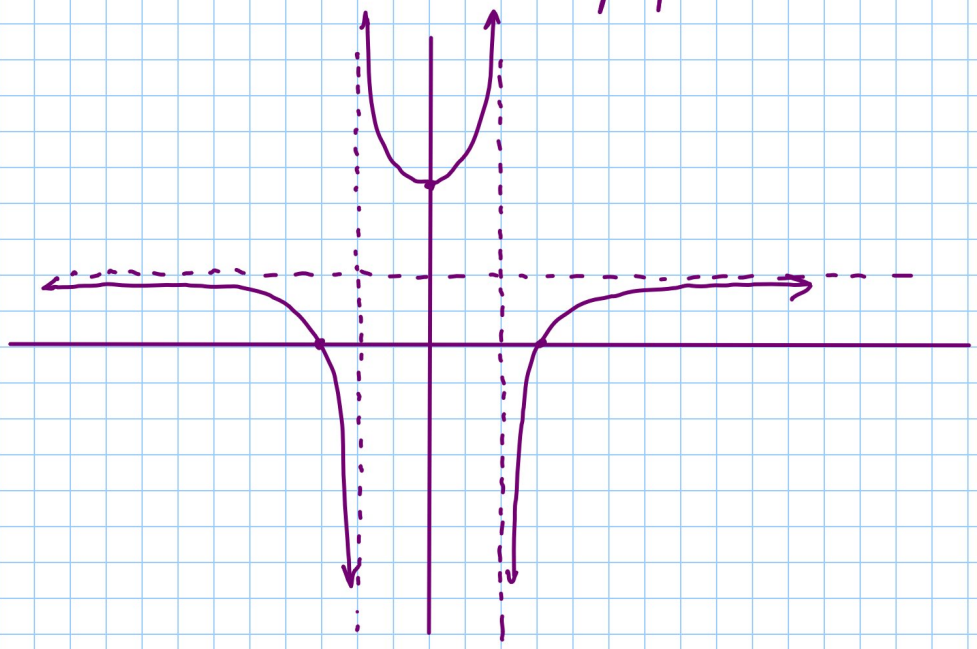
$$-3x^2=4$$

$$x^2=-\frac{4}{3}$$

no real solutions so no PIPs



graph can change concavity around asymptotes



Assignment

Graph $f(x)=\frac{x^2-4}{2x^2-32}$