

Section 6.4 Sum and Difference Formulas

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$

ex: Exact value of $\sin 75^\circ = \sin(30^\circ + 45^\circ) = \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2} + \sqrt{6}}{4}$$

ex: $\tan \frac{\pi}{12} = \tan 15^\circ = \tan(60^\circ - 45^\circ) = \frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \cdot \tan 45^\circ}$

$$\frac{\pi}{12} \cdot \frac{180}{\pi} = 15^\circ$$

$$45^\circ - 30^\circ$$

$$60^\circ - 45^\circ$$

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1} = \frac{(\sqrt{3} - 1)(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)}$$

Conjugate
of $\sqrt{3} + 1$

$$= \frac{3 - 2\sqrt{3} + 1}{3 - 1}$$

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$$= \frac{2 - 2\sqrt{3}}{2} = \frac{2(2 - \sqrt{3})}{2}$$
$$= 2 - \sqrt{3}$$

$$46) \frac{\cos(\alpha - \beta)}{\sin \alpha \cos \beta} = \cot \alpha + \tan \beta$$

$$\frac{\cos(\alpha - \beta)}{\sin \alpha \cos \beta} = \frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\sin \alpha \cos \beta}$$

$$= \frac{\cancel{\cos \alpha} \cancel{\cos \beta}}{\cancel{\sin \alpha} \cancel{\cos \beta}} + \frac{\cancel{\sin \alpha} \sin \beta}{\cancel{\sin \alpha} \cos \beta}$$

$$= \cot \alpha + \tan \beta$$