6) $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

$$\sin\theta(\cot\theta + \tan\theta) = \frac{\sin\theta}{1}\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{\cos\theta}{\cos\theta}\cdot\frac{\cos\theta}{1} + \frac{\sin^2\theta}{\cos\theta} \qquad \text{LCD} = \cos\theta$$

$$= \frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

15) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

$$(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$$

$$= 1 + 1$$

$$= 2$$

Section 6.3 continued

ex: $\csc\theta - \cot\theta = \dfrac{\sin\theta}{1+\cos\theta}$

$$\csc\theta - \cot\theta = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{(1-\cos\theta)}{\sin\theta}\cdot\frac{(1+\cos\theta)}{1+\cos\theta}$$

conjugate of $1-\cos\theta$

In general $a+b$ is the conjugate of $a-b$

$$= \frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin\theta}{1+\cos\theta}$$

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ex: $\frac{\sec\theta}{\csc\theta} + \frac{\sin\theta}{\cos\theta} = 2\tan\theta$

$$\frac{\sec\theta}{\csc\theta} + \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta + \tan\theta$$

$$= 2\tan\theta$$

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21) $3\sin^2\theta + 4\cos^2\theta = 3 + \cos^2\theta$

$$3\sin^2\theta + 4\cos^2\theta = 3(1-\cos^2\theta) + 4\cos^2\theta$$

$$= 3 - 3\cos^2\theta + 4\cos^2\theta$$

$$= 3 + \cos^2\theta$$

24) $1 - \frac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$

$1 - \frac{\sin^2\theta}{1-\cos\theta} = 1 - \frac{1-\cos^2\theta}{1-\cos\theta}$

$= 1 - \frac{(1+\cos\theta)(1-\cos\theta)}{1-\cos\theta}$

$= 1 - 1 - \cos\theta$

$= -\cos\theta$

$\frac{5}{8} \times \frac{2}{7} = \frac{5\cdot 7 + 2\cdot 8}{8\cdot 7} = \frac{35+16}{56} = \frac{51}{56}$

31) $\frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$

$\frac{2}{\cos\theta}$

$\frac{1-\sin\theta}{\cos\theta} \times \frac{\cos\theta}{1-\sin\theta} = \frac{(1-\sin\theta)(1-\sin\theta) + \cos^2\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{1 - 2\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{1 - 2\sin\theta + 1}{\cos\theta(1-\sin\theta)}$

$= \frac{2 - 2\sin\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{2(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$

$= 2\sec\theta$