

$$6) \sin\theta(\cot\theta + \tan\theta) = \sec\theta$$

$$\sin\theta(\cot\theta + \tan\theta) = \frac{\sin\theta}{1} \left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \right)$$

$$= \frac{\cos\theta \cdot \cos\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}$$

$$LCD = \cos\theta$$

$$= \frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

$$15) (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$$

$$(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = \sin^2\theta + \cancel{2\sin\theta\cos\theta} + \cos^2\theta$$

$$+ \sin^2\theta - \cancel{2\sin\theta\cos\theta} + \cos^2\theta$$

$$= 1 + 1$$

$$= 2$$

Section 6.3 continued

$$\text{ex: } \csc\theta - \cot\theta = \frac{\sin\theta}{1 + \cos\theta}$$

$$\csc\theta - \cot\theta = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{(1 - \cos\theta)}{\sin\theta} \cdot \frac{(1 + \cos\theta)}{1 + \cos\theta}$$

conjugate of $1 - \cos\theta$
In general
 $a + b$ is the
conjugate of $a - b$

$$\begin{aligned}
 &= \frac{1 - \cos^2 \theta}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{\sin^2 \theta}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{\sin \theta}{1 + \cos \theta}
 \end{aligned}$$

ex: $\frac{\sec \theta}{\csc \theta} + \frac{\sin \theta}{\cos \theta} = 2 \tan \theta$

$$\begin{aligned}
 \frac{\sec \theta}{\csc \theta} + \frac{\sin \theta}{\cos \theta} &= \frac{\frac{1}{\cos \theta}}{\frac{1}{\sin \theta}} + \frac{\sin \theta}{\cos \theta} \\
 &= \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1} + \frac{\sin \theta}{\cos \theta} \\
 &= \frac{\sin \theta}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \\
 &= \tan \theta + \tan \theta \\
 &= 2 \tan \theta
 \end{aligned}$$

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21) $3 \sin^2 \theta + 4 \cos^2 \theta = 3 + \cos^2 \theta$

$$\begin{aligned}
 \underline{3 \sin^2 \theta} + 4 \cos^2 \theta &= 3(1 - \cos^2 \theta) + 4 \cos^2 \theta \\
 &= 3 - 3 \cos^2 \theta + 4 \cos^2 \theta \\
 &= 3 + \cos^2 \theta
 \end{aligned}$$

$$24) 1 - \frac{\sin^2 \theta}{1 - \cos \theta} = -\cos \theta$$

$$\begin{aligned} 1 - \frac{\sin^2 \theta}{1 - \cos \theta} &= 1 - \frac{1 - \cos^2 \theta}{1 - \cos \theta} \\ &= 1 - \frac{(1 + \cos \theta)(1 - \cos \theta)}{1 - \cos \theta} \\ &= 1 - 1 - \cos \theta \\ &= -\cos \theta \end{aligned}$$

$$\frac{5}{8} \times \frac{2}{7} = \frac{5 \cdot 2}{8 \cdot 7} = \frac{10}{56} = \frac{5}{28}$$

$$31) \frac{1 - \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 - \sin \theta} = 2 \sec \theta \quad \frac{2}{\cos \theta}$$

$$\begin{aligned} \frac{1 - \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 - \sin \theta} &= \frac{(1 - \sin \theta)(1 - \sin \theta) + \cos^2 \theta}{\cos \theta (1 - \sin \theta)} \\ &= \frac{1 - 2\sin \theta + \sin^2 \theta + \cos^2 \theta}{\cos \theta (1 - \sin \theta)} \end{aligned}$$

$$= \frac{1 - 2\sin \theta + 1}{\cos \theta (1 - \sin \theta)}$$

$$= \frac{2 - 2\sin \theta}{\cos \theta (1 - \sin \theta)}$$

$$= \frac{2(1 - \sin \theta)}{\cos \theta (1 - \sin \theta)}$$

$$= 2 \sec \theta$$