Section 3.7 Implicit Differentiation

Today 3.7
Tomorrow - work
Wed. - practest
Thurs - go over
Fri - test

Suppose we wanted to find $\frac{dy}{dx}$ for the equation

$x^2+3y=5$

$$x^2+3y=5$$
$$3y=5-x^2$$
$$y=\frac{5}{3}-\frac{1}{3}x^2$$
$$\frac{dy}{dx}=-\frac{2}{3}x$$

Implicit Differentiation

$\frac{d}{dx}(3f(x))=3f'(x)$

$$x^2+3y=5$$
$$2x+3\cdot\frac{dy}{dx}=0$$

↳ we look at y as a function of x, so the derivative of $3y$ is 3 times the derivative of y, which is $\frac{dy}{dx}$

$$3\frac{dy}{dx}=-2x$$
$$\frac{dy}{dx}=\frac{-2x}{3}$$

ex: $y^3+2y=5x+6x^2$

$$3y^2\cdot\frac{dy}{dx}+2\frac{dy}{dx}=5+12x$$
$$\frac{dy}{dx}(3y^2+2)=5+12x$$
$$\frac{dy}{dx}=\frac{5+12x}{3y^2+2}$$

ex: $5x^3 - 7y^3 = 17x$

$15x^2 - 21y^2 \frac{dy}{dx} = 17$

$-21y^2 \frac{dy}{dx} = 17 - 15x^2$

$\frac{dy}{dx} = \frac{17 - 15x^2}{-21y^2}$

ex: Find the equation of the tangent line to the graph of $3(x^2+y^2)^2 = 100xy$ and the point $(3,1)$.

$6(x^2+y^2)^1 \cdot \left(2x + 2y\frac{dy}{dx}\right) = 100x \cdot \frac{dy}{dx} + y \cdot 100$

$6(3^2+1^2)\cdot\left(2\cdot 3 + 2\cdot 1\frac{dy}{dx}\right) = 100\cdot 3\frac{dy}{dx} + 1\cdot 100$

$6\cdot 10\left(6 + 2\frac{dy}{dx}\right) = 300\frac{dy}{dx} + 100$

$360 + 120\frac{dy}{dx} = 300\frac{dy}{dx} + 100$

$-180\frac{dy}{dx} = -260$

$\frac{dy}{dx} = \frac{-260}{-180} = \frac{13}{9} = m$

$1 = \frac{13}{9}\cdot 3 + b$

$\frac{9}{9} - \frac{39}{9} = b$

$-\frac{30}{9} = b$

$y = \frac{13}{9}x - \frac{30}{9}$

p140 1,2,6,7,18,22

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2) $x^2 + xy - y^3 = xy^2$

$2x + x\frac{dy}{dx} + y\cdot 1 - 3y^2\frac{dy}{dx} = x\cdot 2y\frac{dy}{dx} + y^2\cdot 1$

$x\frac{dy}{dx} - 3y^2\frac{dy}{dx} - 2xy\frac{dy}{dx} = y^2 - 2x - y$

$\frac{dy}{dx}(x - 3y^2 - 2xy) = y^2 - 2x - y$

$$\frac{dy}{dx} = \frac{y^2 - 2x - y}{x - 3y^2 - 2xy}$$

22) $y^2 = \dfrac{x^2}{xy-4}$ $\quad (4,2)$

$y^2(xy-4) = x^2$

$xy^3 - 4y^2 = x^2$

$y = 2$ is tan line

$x \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot 1 - 8y\frac{dy}{dx} = 2x$

$(3xy^2 - 8y)\frac{dy}{dx} = 2x - y^3$

$(3 \cdot 4 \cdot 4 - 8 \cdot 2)\frac{dy}{dx} = 8 - 8$

$\frac{dy}{dx} = 0$