WARMUP

Establish each identity

1) $\sin\theta\csc\theta - \cos^2\theta = \sin^2\theta$

$\sin\theta\csc\theta - \cos^2\theta = \sin\theta \cdot \frac{1}{\sin\theta} - \cos^2\theta$

$= 1 - \cos^2\theta$

$= \sin^2\theta$

$A^2 - B^2 = (A+B)(A-B)$

2) $1 - \frac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$

$1 - \frac{\sin^2\theta}{1-\cos\theta} = 1 - \frac{1-\cos^2\theta}{1-\cos\theta}$

$= 1 - \frac{(1+\cos\theta)\cancel{(1-\cos\theta)}}{\cancel{1-\cos\theta}}$

$= 1 - (1+\cos\theta)$

$= 1 - 1 - \cos\theta$

$= -\cos\theta$

3) $\frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$

$\frac{1-\sin\theta}{\cos\theta} \cancel{+} \frac{\cos\theta}{1-\sin\theta} = \frac{(1-\sin\theta)(1-\sin\theta) + (\cos\theta)(\cos\theta)}{\cos\theta(1-\sin\theta)}$

$= \frac{1 - 2\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1-\sin\theta)}$

$\frac{2}{7} \cancel{+} \frac{3}{5}$

$\frac{2\cdot5 + 3\cdot7}{7\cdot5} = \frac{10+21}{35} = \frac{31}{35}$

$= \frac{1 - 2\sin\theta + 1}{\cos\theta(1-\sin\theta)}$

$= \frac{2 - 2\sin\theta}{\cos\theta(1-\sin\theta)}$

$= \frac{2\cancel{(1-\sin\theta)}}{\cos\theta\cancel{(1-\sin\theta)}}$

$= 2\sec\theta$

1) $\sin\alpha = \frac{3}{5}$ $\underbrace{0<\alpha<\frac{\pi}{2}}_{QI}$

$y=3, r=5,$

$x=4$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$\cos\beta = \frac{40}{41}$ $\underbrace{\frac{3\pi}{2}<\beta<2\pi}_{QIV}$

$x=40, r=41,$

$y=-9$

$\sin\beta = \frac{-9}{41}$

$\cos\beta = \frac{40}{41}$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$= \frac{3}{5}\cdot\frac{40}{41} - \frac{4}{5}\cdot\left(\frac{-9}{41}\right) = \frac{120}{205} + \frac{36}{205} = \frac{156}{205} = \frac{y}{r}$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$= \frac{4}{5}\cdot\frac{40}{41} + \frac{3}{5}\cdot\left(\frac{-9}{41}\right) = \frac{160}{205} - \frac{27}{205} = \frac{133}{205} = \frac{x}{r}$$

$$\tan(\alpha-\beta) = \frac{y}{x} = \frac{156}{133}$$

What Quadrant is $\alpha-\beta$ in? QI

$y>0, x>0$

$133, 156, 205$

$133^2+156^2 = 205^2$

$42025 = 42025$

Find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$ and what Quadrant $\alpha+\beta$ is in.

$\sec\alpha = \frac{-29}{20} = \frac{r}{x}$ $r=29$ $x=-20$

$\frac{\pi}{2}<\alpha<\pi$

$(-20)^2+y^2 = 29^2$

$400+y^2 = 841$

$y^2 = 441$

$y=21$ ← positive QII

$\cos\beta = \frac{24}{25} = \frac{x}{r}$

$\frac{3\pi}{2}<\beta<2\pi$

$x=24$

$r=25$

$y=-7$ ← QIV

$y<0$

$\sin\alpha = \frac{21}{29}$ $\qquad$ $\sin\beta = -\frac{7}{25}$

$\cos\alpha = -\frac{20}{29}$ $\qquad$ $\cos\beta = \frac{24}{25}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{21}{29}\cdot\frac{24}{25} + \left(\frac{-20}{29}\right)\left(-\frac{7}{25}\right) = \frac{504}{725} + \frac{140}{725} = \frac{644}{725} = \frac{y}{r}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= -\frac{20}{29}\cdot\frac{24}{25} - \frac{21}{29}\left(-\frac{7}{25}\right) = \frac{-480}{725} + \frac{147}{725} = \frac{-333}{725} = \frac{x}{r}$

$\tan(\alpha+\beta) = -\frac{644}{333}$

Quadrant II since $x<0$, $y>0$