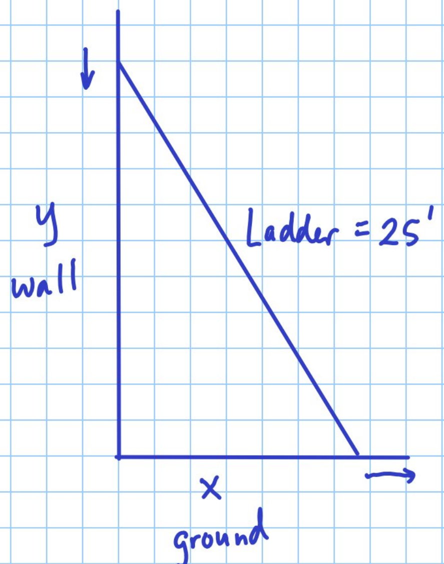
On your desk there is a Larson Calculus book.

For a warmup read pages 153–154.

RELATED RATES

When 2 or more variables are related by an equation and their rates vary with time, they are related rates

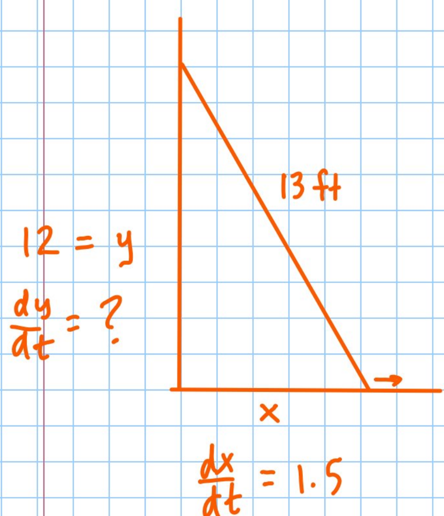


x and y are changing as the ladder slides down the wall.

$$\frac{dx}{dt} = \text{rate of change of } x$$

$$\frac{dy}{dt} = \text{rate of change of } y$$

ex: A 13-foot ladder is leaning against a wall and sliding down. If the foot of the ladder is pulled away from the wall at 1.5 ft/sec, how fast is the top of the ladder sliding down the wall when the top of the ladder is 12 feet above the ground?



STEP 1: Write an equation relating the variables.

$$x^2 + y^2 = 13^2$$

STEP 2: Write the related rate equation by using the derivative.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

have to have these by Chain

Rule since x and y are functions of time.

STEP 3: Sub in what we know and solve for what we don't know.

3, 4, 5
5, 12, 13
7, 24, 25
8, 15, 17
9, 40, 41

$y = 12$

$x^2 + 12^2 = 13^2$

$x^2 + 144 = 169$

$x^2 = 25$

$x = 5$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2 \cdot 5 \cdot 1.5 + 2 \cdot 12 \frac{dy}{dt} = 0$

$15 + 24\frac{dy}{dt} = 0$

$24\frac{dy}{dt} = -15$

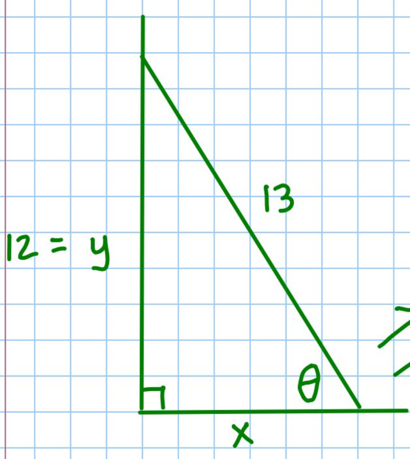
$\frac{dy}{dt} = \frac{-15}{24}$

$\frac{dy}{dt} = \frac{-5}{8}\ \text{ft/sec}$

STEP 4: Answer the question

$-\frac{5}{8}$ ft/sec ← negative because y is getting smaller

ex: Same scenario. How fast is the angle formed by the ladder and the ground changing?



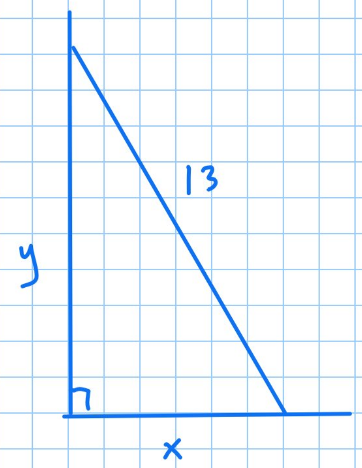
$\cos\theta = \frac{x}{13} = \frac{1}{13}x$ $\frac{dx}{dt} = 1.5 = \frac{3}{2}$

$-\sin\theta \cdot \frac{d\theta}{dt} = \frac{1}{13} \cdot \frac{dx}{dt}$

$\cancel{-\frac{13}{12}}\left(\cancel{-\frac{12}{13}}\right)\frac{d\theta}{dt} = \left(\frac{1}{\cancel{13}} \cdot \frac{\cancel{3}}{2}\right)\left(\frac{-\cancel{13}}{\cancel{12}_{4}}\right)$

$\frac{d\theta}{dt} = -\frac{1}{8}\ \frac{\text{rad}}{\text{sec}}$

ex: Same scenario. At what rate is the area of the Δ formed by the wall, the ground, and ladder changing?



$$A = \frac{1}{2}xy$$

$$\frac{dA}{dt} = \frac{1}{2}x \cdot \frac{dy}{dt} + y \cdot \frac{1}{2}\frac{dx}{dt}$$

$$\frac{dA}{dt} = \frac{1}{2} \cdot 5 \cdot \left(-\frac{5}{8}\right) + 12 \cdot \frac{1}{2} \cdot \frac{3}{2}$$

$$\frac{dA}{dt} = \frac{-25}{16} + \frac{36}{4} = \frac{-25}{16} + \frac{144}{16}$$

$$= \frac{119}{16} \text{ ft}^2/\text{sec}$$

p159 #23 just the 7-ft case

23 *Moving Ladder* A ladder 25 feet long is leaning against the wall of a house (see figure). The base of the ladder is pulled away from the wall at a rate of 2 feet per second.

a. How fast is the top moving down the wall when the base of the ladder is 7 feet, ~~15 feet, and 24 feet~~ from the wall?

b. Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 7 feet from the wall.

c. Find the rate at which the angle between the top of the ladder and the wall of the house is changing when the base of the ladder is 7 feet from the wall.