WARMUP

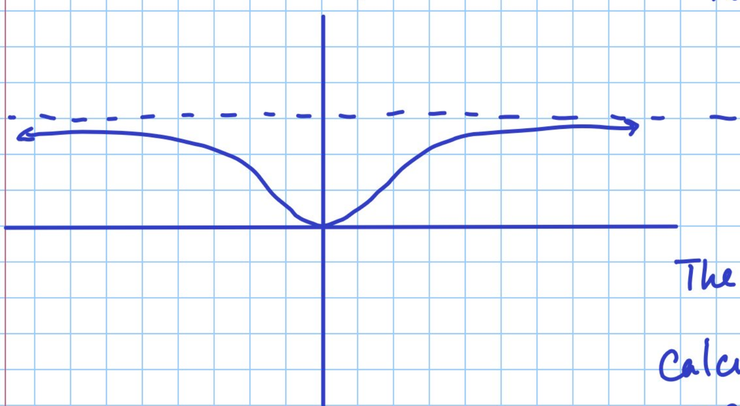
Use Desmos to find horizontal asymptotes

1) $f(x)=\dfrac{3x^2-4}{x^2+5x+4}$ $\quad y=3$

2) $f(x)=\dfrac{\sqrt{4x^2+1}}{x+2}$ $\quad y=2$, $y=-2$

Section 3.5 Limits at Infinity

Consider the graph of $f(x)=\dfrac{3x^2}{x^2+1}$



It has a horizontal asymptote at $y=3$. (Ratio of the leading coefficients)

The H.A.s can be thought of in calculus as Limits at Infinity

$$\lim_{x\to\infty}\frac{3x^2}{x^2+1}=3 \quad\text{and}\quad \lim_{x\to-\infty}\frac{3x^2}{x^2+1}=3$$

In Calculus, instead of memorizing the H.A. rules from Precalc, we multiply top and bottom of the limit by one over the highest power of x in the problem.

ex: $$\lim_{x\to\infty}\frac{3x^2}{(x^2+1)}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}=\lim_{x\to\infty}\frac{3}{1+\frac{1}{x^2}}=\frac{3}{1+0}=3$$

as $x\to\infty$, $\frac{1}{x^2}\to 0$

ex: $$\lim_{x\to-\infty}\left(5-\frac{2}{x^2}\right)=\lim_{x\to-\infty}5-\lim_{x\to-\infty}\frac{2}{x^2}=5-0=5$$

ex: $\lim_{x\to\infty} \frac{(2x^3+5)}{(3x^2+1)} \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{2+\frac{5}{x^3}}{\frac{3}{x}+\frac{1}{x^3}} = \frac{2+0}{0+0} = \frac{2}{0}$

so limit does not exist (no HA)

ex: $\lim_{x\to-\infty} \frac{(3x-2)}{\sqrt{2x^2+1}} \cdot \frac{-\frac{1}{x}}{\sqrt{\frac{1}{x^2}}} = \lim_{x\to-\infty} \frac{-3+\frac{2}{x}}{\sqrt{2+\frac{1}{x^2}}} = \frac{-3+0}{\sqrt{2+0}} = -\frac{3}{\sqrt{2}}$

but $x<0$ so

$\frac{1}{x}$ and $\sqrt{\frac{1}{x^2}}$ are opposites

(plug -5 in you get $\frac{1}{-5}$ and $\sqrt{\frac{1}{(-5)^2}} = \sqrt{\frac{1}{25}} = \frac{1}{5}$)

Assignment
p203 9-23 odd

15) $\lim_{x\to\infty} \left(2x - \frac{1}{x^2}\right) =$ D.N.E.

because $x \to -\infty$

19) $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2-x}} \cdot \frac{-\frac{1}{x}}{\sqrt{\frac{1}{x^2}}} = \lim_{x\to\infty} \frac{-1}{\sqrt{1-\frac{1}{x}}} = \frac{-1}{\sqrt{1-0}}$

$= -1$

59) p195

Cubic polynomial with 3 distinct zeros a, b, c

$f(x) = (x-a)\underbrace{(x-b)(x-c)} = (x-a)(x^2-bx-cx+bc)$

$f(x) = x^3 - bx^2 - cx^2 + bcx - ax^2 + abx + acx - abc$

$f'(x) = 3x^2 - 2bx - 2cx + bc - 2ax + ab + ac$

$f''(x) = 6x - 2b - 2c - 2a = 0$

$6x = 2a + 2b + 2c$

$x = \frac{2a+2b+2c}{6} = \frac{a+b+c}{3}$

23) $$\underbrace{\lim_{x\to\infty}}_{\text{no negatives}} \frac{(x^2-x)}{\sqrt{x^4+1}} \cdot \frac{\frac{1}{x^2}}{\sqrt{\frac{1}{x^4}}} = \lim_{x\to\infty} \frac{1-\frac{1}{x}}{\sqrt{1+\frac{1}{x^4}}} = \frac{1-0}{\sqrt{1+0}} = 1$$