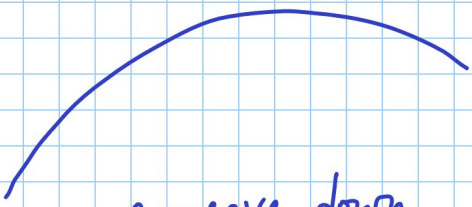
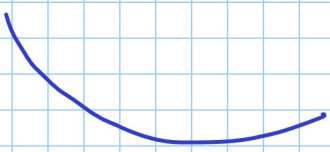
Section 3.4 Concavity and Points of Inflection

Recall:

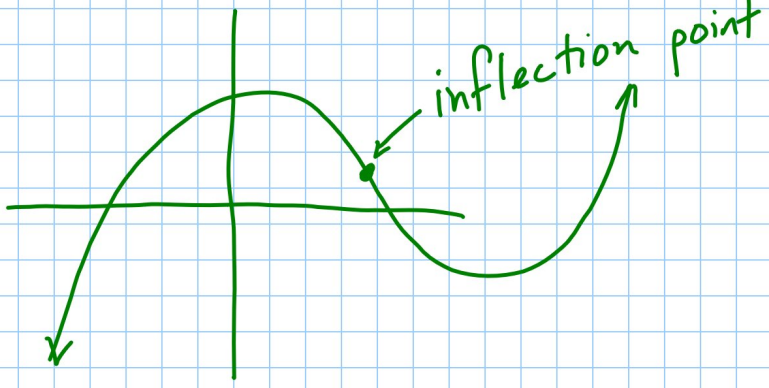


concave down
$f'' < 0$



concave up
$f'' > 0$

A <u>point of inflection</u> is a point where the graph of a function changes concavity.



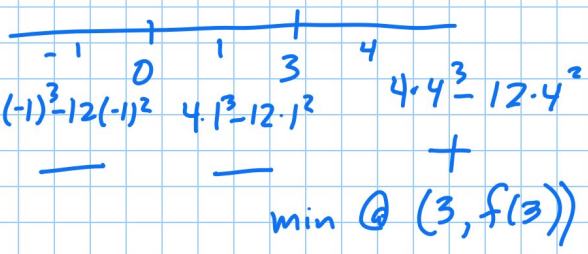
To find inflection points we start by finding possible inflection points (PIPS) by finding domain values for which $f''(x) = 0$ or $f''(x)$ is undefined. Then we use the same number line test we used in the first derivative test.

ex:
Find extrema and points of inflection for $f(x) = x^4 - 4x^3$

Extrema: $f'(x) = 4x^3 - 12x^2$

$4x^2(x-3) = 0$

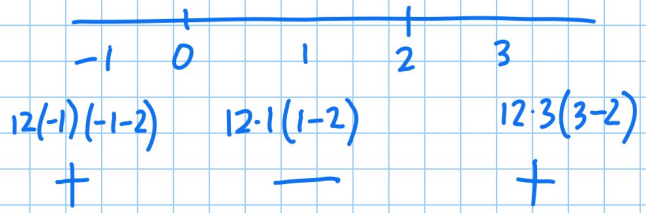
critters are 0, 3



IPs: $f''(x) = 12x^2 - 24x$

$12x(x-2) = 0$

PIPs are $x = 0$, $x = 2$



IPs @ $(0, f(0)) = (0, 0)$

$(2, f(2)) = (2, -16)$

$2^4 - 4 \cdot 2^3$

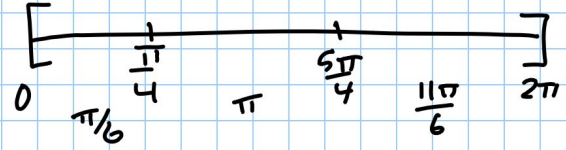
Look at graph on Desmos

ex 34 p 194 $f(x) = \sin x + \cos x$

Extrema: $f'(x) = \cos x - \sin x = 0$

$\cos x = \sin x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$



$+ \quad - \quad +$

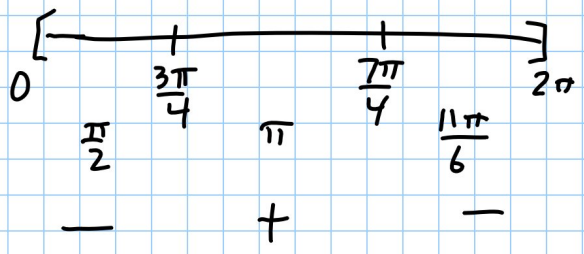
Local max @ $\left(\frac{\pi}{4}, f\left(\frac{\pi}{4}\right)\right) = \left(\frac{\pi}{4}, \sqrt{2}\right)$

Local min @ $\left(\frac{5\pi}{4}, f\left(\frac{5\pi}{4}\right)\right) = \left(\frac{5\pi}{4}, -\sqrt{2}\right)$

IPs: $f''(x) = -\sin x - \cos x$

$-\sin x = \cos x$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

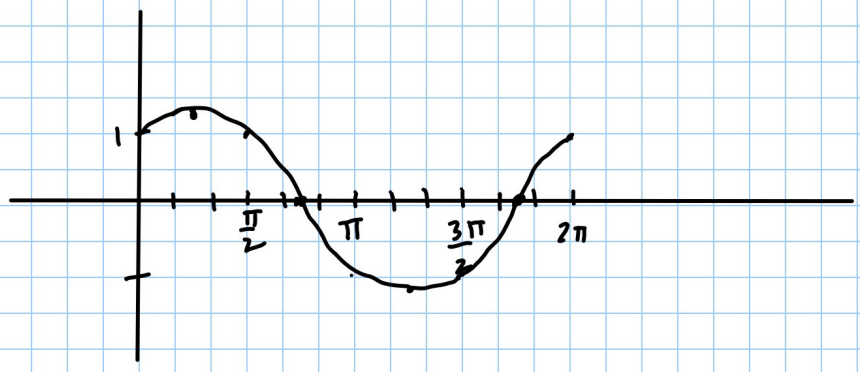


$\frac{\pi}{2} \qquad \pi \qquad \frac{11\pi}{6}$

$- \quad + \quad -$

IPs @ $\left(\frac{3\pi}{4}, f\left(\frac{3\pi}{4}\right)\right) = \left(\frac{3\pi}{4}, 0\right)$

$\left(\frac{7\pi}{4}, f\left(\frac{7\pi}{4}\right)\right) = \left(\frac{7\pi}{4}, 0\right)$



ASSIGNMENT p194-195 21, 23, 25, 27, 43, 55, 59