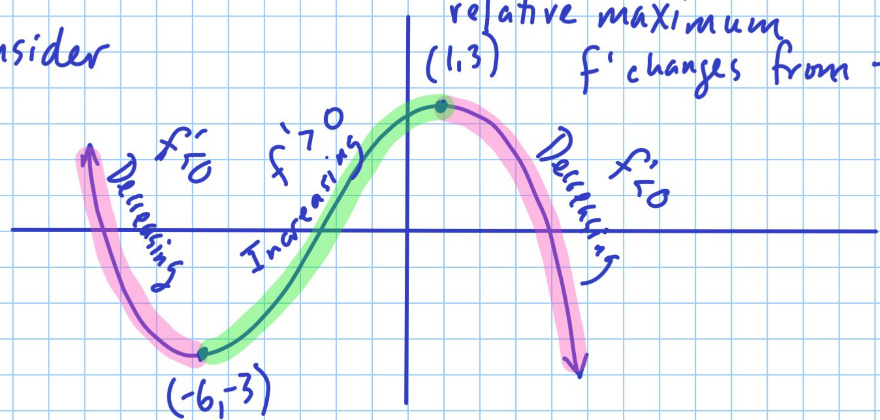
Section 3.3 The First Derivative Test

Consider



relative minimum

f' changes from negative to positive

The First Derivative Test

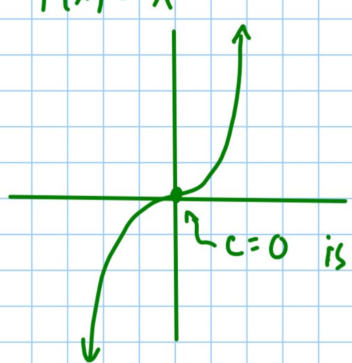
Let c be a critical number of f. [$f'(c)=0$ or $f'(c)$ is undefined]

1) If f' changes from negative to positive at c, then $(c, f(c))$ is a <u>relative minimum</u> of f

2) If f' changes from positive to negative at c, then $(c, f(c))$ is a <u>relative maximum</u>.

There are cases when a critical number does not yield a max or min:

<u>ex</u>: $f(x) = x^3$



c=0 is a critter but not max or min

ex 3 p183 $f(x) = (x^2-4)^{2/3}$

① Find critters: $f'(x) = \frac{2}{3}(x^2-4)^{-1/3} \cdot 2x = \dfrac{4x}{3\sqrt[3]{x^2-4}}$

$f'(x) = 0$ when $4x = 0 \Rightarrow x = 0$

$f'(x) =$ undefined when $3\sqrt[3]{x^2-4} = 0$

$$\left(\sqrt[3]{x^2-4}\right)^3 = 0^3$$

$$x^2-4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

critters $-2, 0, 2$

② Use a number line to determine sign of derivative on intervals determined by critters.

$$\longleftarrow \; -2 \; \cdots \; 0 \; \cdots \; 2 \; \longrightarrow$$

test values plug into derivative to determine + or −

-8	-1	1	3
$\dfrac{4(-8)}{3\sqrt[3]{(-8)^2-4}}$	$\dfrac{4(-1)}{3\sqrt[3]{(-1)^2-4}}$	$\dfrac{4(1)}{3\sqrt[3]{1^2-4}}$	$\dfrac{4(3)}{3\sqrt[3]{3^2-4}}$
$= \dfrac{-}{+}$	$= \dfrac{-}{-}$	$= \dfrac{+}{-}$	$= \dfrac{+}{+}$
$= -$	$= +$	$= -$	$= +$

③ Use first derivative test to determine maxima and minima.

f' from − to + at $x = -2$, so $(-2, f(-2)) = (-2, 0)$ is min

$((-2)^2-4)^{2/3}$

f' from + to − at $x = 0$, so $(0, f(0)) = (0, 2.52)$ is max

f' from − to + at $x = 2$, so $(2, f(2)) = (2, 0)$ is min

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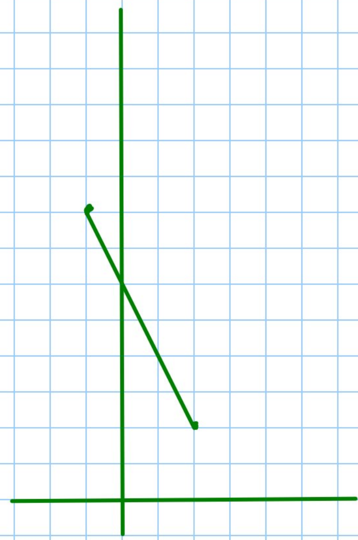
$f(x) = 2(3-x)$ on $[-1, 2]$

$= 6-2x$

$f'(x) = -2$ no critics

$f(-1) = 2(4) = 8 \leftarrow$ max

$f(2) = 2(1) = 2 \leftarrow$ min



44) $C = 2x + \frac{300000}{x} = 2x + 300000x^{-1}$ $[1, 300]$

$C' = 2 - 300000x^{-2} = 2 - \frac{300000}{x^2} = 0$

$-\frac{300000}{x^2} = -2$

$300000 = 2x^2$

$150000 = x^2$

$x \approx 387$

$C(1) = 2 + \frac{300000}{1} = 300{,}002$

$C(300) = 2 \cdot 300 + \frac{3000\cancel{00}}{3\cancel{00}} = 600 + 1000 = 1600$