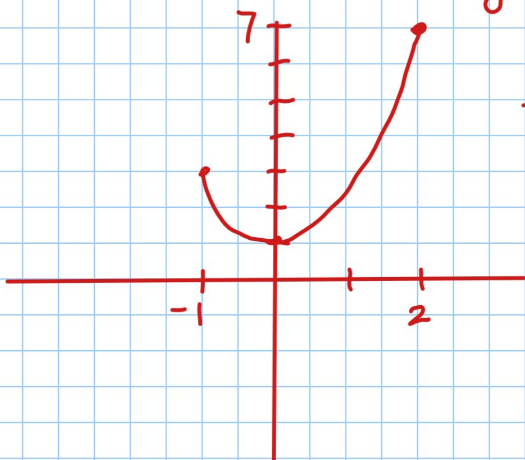
Section 3.1 Extrema on an Interval

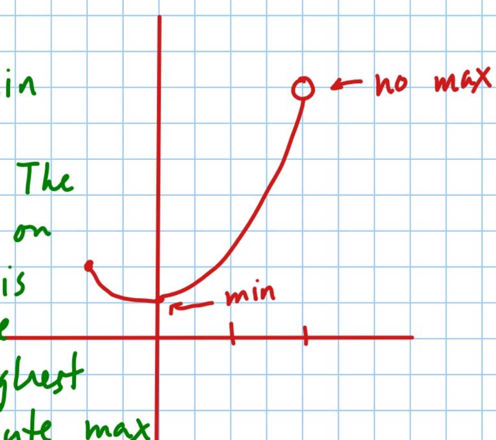
Consider the following graph on the interval $[-1, 2]$

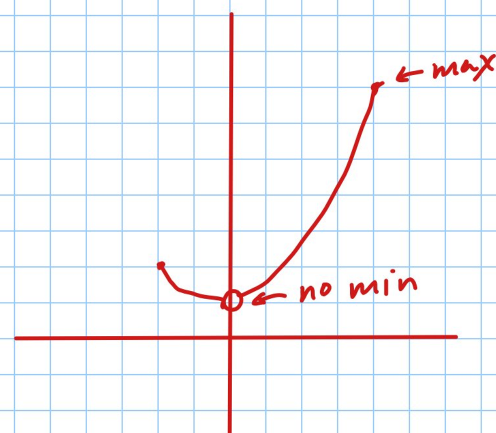


This graph has a minimum at $(0,1)$ since it has the lowest y-coordinate of any point.

Likewise, the graph has a maximum at $(2,7)$ since it has the highest y-coordinate of any point.

A max or min is called an extremum. The lowest point on an interval is the absolute min, the highest is the absolute max

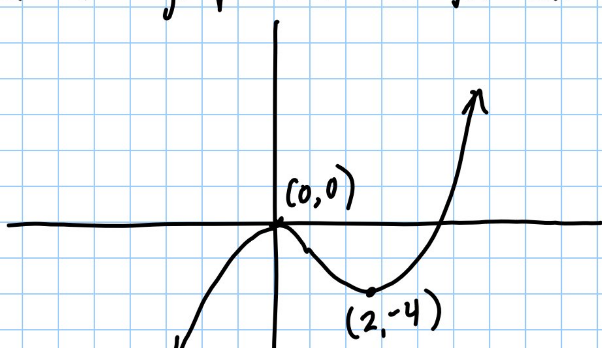




If $f(x)$ is continuous on the closed interval $[a, b]$ then f has a min and a max on that interval.

Extrema on a closed interval must happen at an endpoint, or at $x = c$ such that $f'(c) = 0$ or $f'(c)$ is undefined. Such a c is called a <u>critical number</u> of f

Consider the graph defined for all real numbers



$(0,0)$ is called a <u>relative or local max</u>

$(2,-4)$ is called a <u>relative or local min</u>

These are the max and min points in a local neighborhood.

To find Extrema on a Closed Interval $[a,b]$

1) Find the critical numbers in (a,b)
2) Evaluate f at those critical numbers.
3) Evaluate f at each endpoint.
4) The greatest of these numbers is the max.
   The least of these numbers is the min.

ex 2 p168

$f(x) = 3x^4 - 4x^3$ on $[-1,2]$

1) $f'(x) = 12x^3 - 12x^2$

$= 12x(x-1)$

$12x = 0 \qquad x - 1 = 0$

$x = 0 \qquad x = 1$

2) $f(0) = 3\cdot 0^4 - 4\cdot 0^3 = 0$

$f(1) = 3\cdot 1^4 - 4\cdot 1^3 = -1 \Rightarrow$

4 $\boxed{f(1) = -1 \text{ is min} \quad f(2) = 16 \text{ is max}}$

3) $f(-1) = 3(-1)^4 - 4(-1)^3$

$= 3 + 4 = 7$

$f(2) = 3\cdot 2^4 - 4\cdot 2^3$

$= 48 - 32 = 16$

ex3 p169

$f(x) = 2x - 3x^{2/3}$ on $[-1,3]$

$f'(x) = 2 - 2x^{-1/3} = 2 - \frac{2}{x^{1/3}}$ undefined when $x = 0$

$2 - \frac{2}{x^{1/3}} = 0$

$-\frac{2}{x^{1/3}} = -2$

$-2 = -2x^{1/3}$

$1 = x^{1/3}$ $\quad f'(x) = 0$ when $x = 1$

$1 = x$

use calculator

$f(0) = 0 \longleftarrow$ max

$f(1) = -1$

$f(-1) = -5 \longleftarrow$ min

$f(3) = -0.24$

Assignment

p170-171 7, 9, 13, 17, 23, 29, 31, 43, 44