

$$1) \quad \sin \alpha = \frac{-8}{17} \quad \sin \beta = \frac{-9}{41}$$

$$\cos \alpha = \frac{-15}{17} \quad \cos \beta = \frac{40}{41}$$

$$1) \quad \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$= \left(\frac{-8}{17}\right)\left(\frac{40}{41}\right) + \left(\frac{-15}{17}\right)\left(\frac{-9}{41}\right)$$

$$= \frac{-320}{697} + \frac{135}{697} = \frac{-185}{697} = \frac{y}{r}$$

$$3) \quad \tan(\alpha + \beta) = \frac{-185}{-672} = \frac{185}{672}$$

$$2) \quad \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \left(\frac{-15}{17}\right)\left(\frac{40}{41}\right) - \left(\frac{-8}{17}\right)\left(\frac{-9}{41}\right)$$

$$= \frac{-600}{697} - \frac{72}{697} = \frac{-672}{697} = \frac{x}{r}$$

4) Q III since
 $x < 0, y < 0$

PPP $\sec \alpha = -\frac{17}{8} \quad \pi < \alpha < \frac{3\pi}{2}$

$\cot \beta = \frac{5}{12} \quad 0 < \beta < \frac{\pi}{2}$

$\sin(\alpha - \beta)$

$\cos(\alpha - \beta)$

$\tan(\alpha - \beta)$

Quadrant $\alpha - \beta$

$$5) \quad \sin 112.5^\circ = \sin \frac{225^\circ}{2} = + \sqrt{\frac{1 - \cos 225^\circ}{2}} = \sqrt{\frac{1 - (-\sqrt{2}/2)}{2}}$$

Q II
 $\sin \theta > 0$
in Q II

$$= \sqrt{\frac{(1 + \sqrt{2}/2)}{2}} \cdot \frac{2}{2}$$

$$= \sqrt{\frac{2 + \sqrt{2}}{4}}$$

$$= \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$6) \underbrace{\cos 165^\circ}_{\text{QII}} = \cos \frac{330^\circ}{2} = -\sqrt{\frac{1 + \cos 330^\circ}{2}} = -\sqrt{\frac{(1 + \sqrt{3}/2) \cdot 2}{2}}$$

$$= -\sqrt{\frac{2 + \sqrt{3}}{4}}$$

$$= -\frac{\sqrt{2 + \sqrt{3}}}{2}$$

so $\cos \theta < 0$

PPP half-angle

5) $\cos 105^\circ$

6) $\sin 15^\circ$

7-10

$$\cot \theta = \frac{24}{7} \quad 0 < \theta < \frac{\pi}{2}$$

$$x=24 \quad y=7, \quad r=25$$

$$\sin \theta = \frac{7}{25}$$

$$\cos \theta = \frac{24}{25}$$

$$7) \sin(2\theta) = 2 \sin \theta \cos \theta$$

$$= 2 \cdot \frac{7}{25} \cdot \frac{24}{25}$$

$$= \frac{336}{625} = \frac{y}{r}$$

$$8) \cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= \left(\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2$$

$$= \frac{576}{625} - \frac{49}{625}$$

$$= \frac{527}{625} = \frac{x}{r}$$

$$9) \tan(2\theta) = \frac{336}{527}$$

PPP

$$\csc \theta = \frac{41}{9} \quad 0 < \theta < \frac{\pi}{2}$$

$$\sin(2\theta), \cos(2\theta), \tan(2\theta)$$

$$\tan \frac{\theta}{2}$$

$$10) \tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\left(1 - \frac{24}{25}\right) \cdot 25}{\frac{7}{25} \cdot 25}$$

$$= \frac{25 - 24}{7} = \frac{1}{7}$$

$$11) \cos^2 \theta (1 + \tan^2 \theta) = 1$$

$$\cos^2 \theta (1 + \tan^2 \theta) = \cos^2 \theta \cdot \sec^2 \theta$$

$$= \cos^2 \theta \cdot \frac{1}{\cos^2 \theta}$$

$$= 1$$

PPP

$$\sin^2 \theta (1 + \cot^2 \theta) = 1$$

$$12) \frac{\sin^2 \theta}{1 - \cos \theta} - 1 = \cos \theta$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\frac{\sin^2 \theta}{1 - \cos \theta} - 1 = \frac{1 - \cos^2 \theta}{1 - \cos \theta} - 1$$

$$= \frac{(1 + \cos \theta)(\cancel{1 - \cos \theta})}{\cancel{1 - \cos \theta}} - 1$$

$$= 1 + \cos \theta - 1$$

$$= \cos \theta$$

PPP

$$1 - \frac{\cos^2 \theta}{1 + \sin \theta} = \sin \theta$$

$$13) 4 \cos^2 \theta = 3$$

$$\cos^2 \theta = \frac{3}{4}$$

$$\cos \theta = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

PPP

$$4 \sin^2 \theta = 3$$

$$14) 2 \sin^2 \theta - 3 \sin \theta + 1 = 0$$

$$(2 \sin \theta - 1)(\sin \theta - 1) = 0$$

$$2 \sin \theta - 1 = 0$$

$$\sin \theta = \frac{1}{2}$$

$$\sin \theta - 1 = 0$$

$$\sin \theta = 1$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$$

PPP

$$2 \cos^2 \theta - \cos \theta - 1 = 0$$

$$15) \sin^{-1} \frac{\sqrt{2}}{2} = \frac{\pi}{4}$$

PPP 15) $\tan^{-1} \sqrt{3}$

$$16) \sin^{-1} \left(\sin \frac{5\pi}{3} \right) = \sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) = -\frac{\pi}{3}$$

$$16) \sin^{-1} \left(\sin \frac{11\pi}{6} \right)$$

$$17) \cot \left(\sin^{-1} \frac{24}{25} \right) = \frac{7}{24}$$

↑
QI
 $y=24, r=25, x=7$

PPP
17) $\sec \left(\sin^{-1} \frac{8}{17} \right)$

$$18) \sec \left(\sin^{-1} \left(-\frac{40}{41} \right) \right) = \frac{41}{9}$$

↑
QIV
 $y=-40, r=41, x=9$

$$18) \cot \left(\tan^{-1} \left(-\frac{24}{7} \right) \right)$$

$$19) \csc^{-1}(2) = \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$$

$$\csc^{-1} N = \sin^{-1} \frac{1}{N}$$

PPP

$$19) \sec^{-1}(2)$$

$$\sec^{-1} N = \cos^{-1} \frac{1}{N}$$

$$20) \tan^{-1}(-1)$$

$$20) \cot^{-1}(1) = \tan^{-1}(1) = \frac{\pi}{4}$$

$$\cot^{-1} N = \tan^{-1} \frac{1}{N}$$