Find $f'(x)$ if $f(x) = (3x^2+2)^2 = (3x^2+2)(3x^2+2)$

$$f(x) = 9x^4 + 12x^2 + 4$$

$$f'(x) = 36x^3 + 24x = 12x(3x^2+2)$$

Now do $\underbrace{f(x) = (3x^2+2)^{19}}$

Section 3.4 The Chain Rule

Consider the function $f(x) = (3x^2+2)^2$. We could calculate $f'(x)$ like we did in the warmup. Or we could use the Chain Rule.

ex: $f(x) = (3x^2+2)^2$

$$f'(x) = \underbrace{2(3x^2+2)^1}_{\text{POWER RULE}} \cdot \underbrace{(6x)}_{\text{deriv. of what's in parentheses}}$$

$$f'(x) = 12x(3x^2+2)$$

ex: $f(x) = (3x^2+2)^{19}$

$$f'(x) = 19(3x^2+2)^{18} \cdot 6x$$

$$f'(x) = 114x(3x^2+2)^{18}$$

CHAIN RULE

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

ex: $w = (x^3+1)^{100}$

$$w' = 100(x^3+1)^{99} \cdot 3x^2$$

$$w' = 300x^2(x^3+1)^{99}$$

ex: $f(x) = \sqrt{x^4+1} = (x^4+1)^{1/2}$

$$f'(x) = \frac{1}{2}(x^4+1)^{-1/2} \cdot 4x^3$$

$$f'(x) = \frac{2x^3}{(x^4+1)^{1/2}} = \frac{2x^3}{\sqrt{x^4+1}}$$

ex: $f(x) = e^{2x}$

$$f'(x) = e^{2x} \cdot 2$$

$$f'(x) = 2e^{2x}$$

In general

$$\frac{d}{dx}\left[e^{kx}\right] = ke^{kx}$$

ex: $g(x) = 3^{7x^3+2x}$

$$g'(x) = 3^{7x^3+2x} \cdot \ln 3 \cdot (21x^2+2)$$

ex: $f(x) = \underline{x}\,\underline{e^{5-2x}}$

$$f'(x) = \underset{\substack{\uparrow \\ \text{1st}}}{x}\underbrace{e^{5-2x} \cdot (-2)}_{\text{deriv. of 2nd}} + \underset{\substack{\uparrow \\ \text{2nd}}}{e^{5-2x}} \cdot \underset{\substack{\uparrow \\ \text{deriv of 1st}}}{1}$$

$$f'(x) = e^{5-2x}(-2x+1)$$

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1} \qquad \frac{d}{dx}\left[(f(x))^n\right] = n(f(x))^{n-1} \cdot f'(x)$$

$$\frac{d}{dx}\left[e^x\right] = e^x \qquad \frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} \cdot f'(x)$$

$$\frac{d}{dx}\left[a^x\right] = a^x \ln a \qquad \frac{d}{dx}\left[a^{f(x)}\right] = a^{f(x)} \cdot \ln a \cdot f'(x)$$

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every other odd

5) $w = (\sqrt{t}+1)^{100} = (t^{\frac{1}{2}}+1)^{100}$

$w' = 100(t^{\frac{1}{2}}+1)^{99} \cdot \frac{1}{2} t^{-1/2}$

$w' = \dfrac{50(\sqrt{t}+1)^{99}}{\sqrt{t}}$

17) $g(t) = e^{(1+3t)^2}$

$g(t) = e^{(1+6t+9t^2)}$

$g'(t) = e^{(1+6t+9t^2)} \cdot (6+18t)$

$g'(t) = e^{(1+3t)^2} \cdot 2(1+3t)^1 \cdot 3$

$= 6(1+3t)e^{(1+3t)^2}$

21) $y = e^{\frac{3}{2}w}$

25) $y = te^{-t^2}$

$y' = te^{-t^2} \cdot (-2t) + e^{-t^2} \cdot 1$

$y' = e^{-t^2}(-2t^2 + 1)$

33) $y = \dfrac{1}{e^{3x}+x^2} = (e^{3x}+x^2)^{-1}$

$y' = -1(e^{3x}+x^2)^{-2} \cdot (3e^{3x}+2x)$

$y' = \dfrac{-3e^{3x} - 2x}{(e^{3x}+x^2)^2}$

37) $f(\theta) = \dfrac{1}{1+e^{-\theta}} = \left(1+e^{-\theta}\right)^{-1}$

$f'(\theta) = -1\left(1+e^{-\theta}\right)^{-2} \cdot e^{-\theta}(-1)$

$= \dfrac{e^{-\theta}}{\left(1+e^{-\theta}\right)^{2}}$

$(a^m)^n = a^{mn}$

41) $f(y) = \sqrt{10^{(5-y)}} = \left[10^{(5-y)}\right]^{\frac{1}{2}} = 10^{5/2 - 1/2\, y}$

$f'(y) = 10^{5/2 - 1/2\, y} \cdot \ln 10 \cdot \left(-\frac{1}{2}\right) = \dfrac{-\sqrt{10^{(5-y)}} \cdot \ln 2}{2}$