WARMUP

Find $f'(x)$

1) $f(x) = 5e^x - 7x^2 + 8e^{\pi}$ $\qquad f'(x) = 5e^x - 14x$

2) $f(x) = 5 \cdot 3^x$ $\qquad f'(x) = 5 \cdot 3^x \ln 3$

3) $f(x) = (6x-1)(7x+5) = 42x^2 + 23x - 5$

$f'(x) = 84x + 23$

Section 3.3 The Product and Quotient Rules

PRODUCT RULE: $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

$$\frac{d}{dx}[f(x) \cdot g(x)] = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) + f(x+h)g(x) - f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \left(\frac{f(x+h)g(x+h) - f(x+h)g(x)}{h} + \frac{f(x+h)g(x) - f(x)g(x)}{h} \right)$$

$$= \lim_{h \to 0} \left[\frac{f(x+h)}{1} \left(\frac{g(x+h) - g(x)}{h} \right) + \frac{g(x)}{1} \left(\frac{f(x+h) - f(x)}{h} \right) \right]$$

$$= \lim_{h \to 0} f(x+h) \cdot \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} + \lim_{h \to 0} g(x) \cdot \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}\underbrace{[f(x) \cdot g(x)]}_{1^{st} \cdot 2^{nd}} = \underbrace{f(x)}_{1^{st}} \cdot \underbrace{g'(x)}_{\text{derivative of } 2^{nd}} + \underbrace{g(x)}_{2^{nd}} \cdot \underbrace{f'(x)}_{\text{derivative of } 1^{st}}$$

ex: $y = \underbrace{(x^2+3)}_{1st}\underbrace{e^x}_{2nd}$

$y' = (x^2+3)\cdot e^x + e^x \cdot 2x$

$y' = e^x(x^2+3+2x)$

$y' = e^x(x^2+2x+3)$

ex: $f(x) = (x^2+\sqrt{x})(x^2-\sqrt{x})$

$f(x) = x^4 - x$

$f'(x) = 4x^3 - 1$

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

$$= \frac{\text{bot}\cdot \text{der top} - \text{top}\cdot \text{der bot}}{\text{bot}^2}$$

ex: $g(x) = \frac{25x^2}{e^x}$ $\frac{\text{top}}{\text{bot}}$

$$g'(x) = \frac{e^x\cdot 50x - 25x^2\cdot e^x}{(e^x)^2}$$

GCF $= 25xe^x$

$$g'(x) = \frac{25xe^x(2-x)}{(e^x)^2}$$

$$g'(x) = \frac{25x(2-x)}{e^x}$$

ex: $y = \dfrac{x^2+5x+2}{x+3}$

$$y' = \frac{(x+3)(2x+5) - (x^2+5x+2)\cdot 1}{(x+3)^2}$$

$$y' = \frac{2x^2+11x+15 - x^2-5x-2}{(x+3)^2}$$

$$y' = \frac{x^2+6x+13}{(x+3)^2}$$

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3, 4, 9, 10, 13, 14, 27, 29