WARMUP

1) Given $f(x) = x^2 - 3x$, calculate

$$\frac{f(3+h) - f(3)}{h} = \frac{\left[(3+h)^2 - 3(3+h)\right] - \left[3^2 - 3\cdot 3\right]}{h} = \frac{(\cancel{9} + 6h + h^2 - \cancel{9} - 3h) - 0}{h}$$

$$= \frac{h^2 + 3h}{h}$$

$$= \frac{\cancel{h}(h+3)}{\cancel{h}} = h+3$$

2) $$\lim_{h\to 0} \frac{\left(\sqrt{10+h} - \sqrt{10}\right)}{h} \cdot \frac{\left(\sqrt{10+h} + \sqrt{10}\right)}{\left(\sqrt{10+h} + \sqrt{10}\right)}$$

$$= \lim_{h\to 0} \frac{\overset{1}{\cancel{10} + \cancel{h} - \cancel{10}}}{\cancel{h}\left(\sqrt{10+h} + \sqrt{10}\right)}$$

$$= \frac{1}{\sqrt{10+0} + \sqrt{10}}$$

$$= \frac{1}{2\sqrt{10}}$$

Section 2.3 The Derivative at a Point

$$\text{Average Rate of Change of } f \text{ on } [a, a+h] = \frac{f(a+h) - f(a)}{h}$$

The derivative of f at a written $f'(a)$ ("f prime of a")

is defined as

$$\text{Rate of change of } f \text{ at } a = \text{Slope of tangent line at } (a, f(a)) = f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

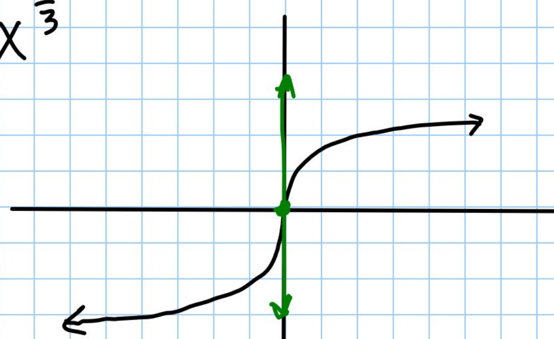
★ Rate of Change $\Longleftrightarrow$ Slope of tangent line $\Longleftrightarrow$ Derivative

When you are asked to calculate $f'(a)$ using the definition of derivative you must use

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If $\displaystyle\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ exists, we say f is <u>differentiable</u> at $x = a$. The process of calculating the derivative is called <u>differentiation</u>.

ex: $f(x) = x^{\frac{1}{3}}$



Not differentiable at $x = 0$ since there's a vertical tangent.

ex: Calculate $f'(-2)$ if $f(x) = 2x^2 + 3x - 4$ using definition of derivative.

$$f'(-2) = \lim_{h \to 0} \frac{\overbrace{f(-2+h)} - \overbrace{f(-2)}}{h}$$

$$= \lim_{h \to 0} \frac{\left[2\overbrace{(-2+h)^2 + 3(-2+h) - 4}\right] - \left[\overbrace{2(-2)^2 + 3(-2) - 4}\right]}{h}$$

$$= \lim_{h \to 0} \frac{\left[2(4 - 4h + h^2) - 6 + 3h - 4\right] - \left[\overbrace{2 \cdot 4 - 6 - 4}^{8-6-4}\right]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{8} - 8h + 2h^2 - \cancel{6} + 3h - \cancel{4} - \cancel{(-2)}}{h}$$

$$= \lim_{h \to 0} \frac{2h^2 - 5h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2h-5)}{\cancel{h}}$$

$$= 2\cdot 0 - 5$$

$$= -5$$

ex: $f'(9)$ if $f(x)=\sqrt{x}$

$$f'(9) = \lim_{h\to 0} \frac{f(9+h)-f(9)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{9+h}-\sqrt{9}}{h}$$

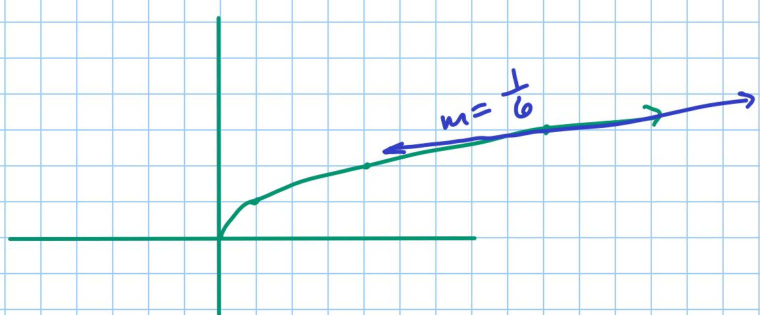
$$= \lim_{h\to 0} \frac{(\sqrt{9+h}-3)}{h} \frac{(\sqrt{9+h}+3)}{(\sqrt{9+h}+3)}$$

$$= \lim_{h\to 0} \frac{\cancel{9}+\cancel{h}^{1}-\cancel{9}}{\cancel{h}(\sqrt{9+h}+3)}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3}$$

$$= \frac{1}{6}$$



Assignment

Using definition of derivative find:

1) $f'(-3)$ if $f(x) = 5x^2 - 4$

2) $f'(16)$ if $f(x) = 2\sqrt{x}$