Math 251 Practice Test 1

Name: KEY

Complete each of the following problems. Show all necessary work.

1. Given $f(x)=\begin{cases}3x-4 & x<-1\\ x^2-8 & x>-1\end{cases}$ find:

a. $\lim_{x\to -1^-} f(x) =$ $3(-1)-4 = -3-4 = -7$

b. $\lim_{x\to -1^+} f(x) =$ $(-1)^2-8 = 1-8 = -7$

c. $\lim_{x\to -1} f(x) =$ -7

2. Find the average velocity for the position function $s(t)=6t-t^3$, in cm, over the interval $0 \le t \le 2$, where t is in seconds. Be sure to state the units in your answer. (cm/sec)

$$\frac{s(2)-s(0)}{2-0} = \frac{(6\cdot 2-2^3)-(6\cdot 0-0^3)}{2}$$

$$= \frac{12-8-0}{2} = \frac{4}{2} = 2 \text{ cm/sec}$$

3. Use the definition of derivative $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ to calculate $f'(16)$ if $f(x)=\sqrt{x}$.

$$f'(16) = \lim_{h\to 0} \frac{(\sqrt{16+h}-\sqrt{16})}{h}\frac{(\sqrt{16+h}+\sqrt{16})}{(\sqrt{16+h}+\sqrt{16})}$$

$$= \lim_{h\to 0} \frac{\cancel{16}+\cancel{h}-\cancel{16}}{\cancel{h}(\sqrt{16+h}+4)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{16+h}+4}$$

$$= \frac{1}{\sqrt{16}+4} = \frac{1}{4+4} = \frac{1}{8}$$

4. Use the definition of derivative $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ if $f(x)=7x-x^2$.

$$f'(x)=\lim_{h\to 0}\frac{\left[7(x+h)-(x+h)^2\right]-\left[7x-x^2\right]}{h}$$

$$=\lim_{h\to 0}\frac{\left[7x+7h-(x^2+2xh+h^2)\right]-\left[7x-x^2\right]}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{7x}+7h-\cancel{x^2}-2xh-h^2-\cancel{7x}+\cancel{x^2}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(7-2x-h)}{\cancel{h}}$$

$$=7-2x$$

5. Find the equation of the tangent line to the graph of $f(x)=8x^3-10x^2+5$ when $x=-2$. You may use the shortcut for this one.

$$f'(x)=24x^2-20x$$

$$m=f'(-2)=24(-2)^2-20(-2)$$

$$=24\cdot 4+40$$

$$=96+40$$

$$=136$$

$\frac{d}{dx}[kx^n]=nkx^{n-1}$

$\frac{d}{dx}[kx]=k$

$\frac{d}{dx}[k]=0$

$$\text{Point}=(-2,f(-2))=(-2,-99)$$

$$f(-2)=8(-2)^3-10(-2)^2+5$$

$$=8(-8)-10\cdot 4+5$$

$$=-64-40+5=-99$$

$$-99=136(-2)+b$$

$$-99=-272+b$$

$$173=b$$

$$y=136x+173$$

In 6-8 evaluate each limit.

6. $\lim_{h\to 0}\dfrac{\frac{7}{3+h}-\frac{7}{3}}{h}$ $= \lim_{h\to 0}\dfrac{21-7(3+h)}{(3+h)\cdot 3h}$

$= \lim_{h\to 0}\dfrac{\cancel{21}-\cancel{21}-7\cancel{h}}{(3+h)\cdot 3\cancel{h}}$

$= \lim_{h\to 0}\dfrac{-7}{(3+h)\cdot 3} = \dfrac{-7}{3\cdot 3} = -\dfrac{7}{9}$

7. $\lim_{h\to 0}\dfrac{(7h+3)^2-9}{h}$ $= \lim_{h\to 0}\dfrac{49h^2+42h+\cancel{9}-\cancel{9}}{h}$

$= \lim_{h\to 0}\dfrac{\cancel{h}(49h+42)}{\cancel{h}} = 49\cdot 0+42$

$= 42$

8. $\lim_{x\to 8}\dfrac{x-8}{x^2-12x+32}$ $= \lim_{x\to 8}\dfrac{\cancel{(x-8)}^{1}}{\cancel{(x-8)}(x-4)} = \dfrac{1}{8-4} = \dfrac{1}{4}$

9. A caterer charges a delivery fee of $25 plus $11.25 per person for a meal.

   a. Write how much the caterer charges, *C*, as a function of number of meals served, *m*.

   b. How much would you pay if you ordered meals for 56 people?

   c. You have $925 budgeted for catering. How many people can be served?

a) $C = 25 + 11.25m$

b) $C = 25 + 11.25 \cdot 56 = \$655$

c) $925 = 25 + 11.25m$

$900 = 11.25m$

$m = 80$ people

10. Given the following graph of $f(x)$, tell whether $f(x)$, $f'(x)$, and $f''(x)$ are positive, negative, or zero at the indicated points. You can use symbols +, -, or 0 instead of the words.

	$f(x)$	$f'(x)$	$f''(x)$
A	-	+	-
B	+	0	-
C	-	-	+
D	-	0	0
E	+	+	+