

Section 7.3 The Law of Cosines

Use when SSS or SAS

For SAS:

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

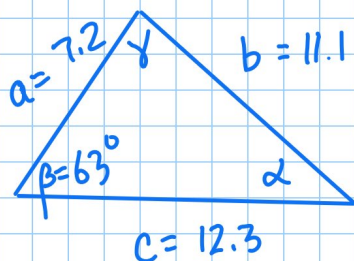
For SSS:

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos \beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

ex: Solve the triangle:



$$\begin{aligned} b &= 11.1 \\ \alpha &= 36.1^\circ \\ \gamma &= 80.9^\circ \end{aligned}$$

$$\frac{\sin \gamma}{12.3} = \frac{\sin 63^\circ}{11.1}$$

$$\sin \gamma = \frac{12.3 \sin 63^\circ}{11.1}$$

$$\sin \gamma = .9873$$

$$\gamma = \sin^{-1} .9873$$

$$\gamma = 80.9^\circ$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$b^2 = 7.2^2 + 12.3^2 - 2 \cdot 7.2 \cdot 12.3 \cos 63^\circ$$

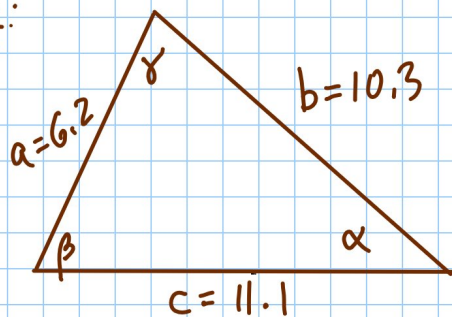
$$b^2 = 122.72$$

$$b = 11.1$$

$$\alpha = 180^\circ - 80.9^\circ - 63^\circ$$

$$\alpha = 36.1^\circ$$

ex:



In SSS ones, always find the biggest angle first.

$$\alpha = 33.4^\circ$$

$$\beta = 66.2^\circ$$

$$\gamma = 80.4^\circ$$

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab} = \frac{(6.2^2 + 10.3^2 - 11.1^2)}{(2 \cdot 6.2 \cdot 10.3)} = 0.1669$$

$$\gamma = \cos^{-1} 0.1669 = 80.4^\circ$$

$$\cos \beta = \frac{a^2 + c^2 - b^2}{2ac} = \frac{(6.2^2 + 11.1^2 - 10.3^2)}{(2 \cdot 6.2 \cdot 11.1)} = 0.4037$$

$$\beta = \cos^{-1} 0.4037 = 66.2^\circ$$

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