WARMUP

In your notebook:

Factor:

1) x^2-49 $\quad (x+7)(x-7)$

2) x^2-7x-8 $\quad (x-8)(x+1)$

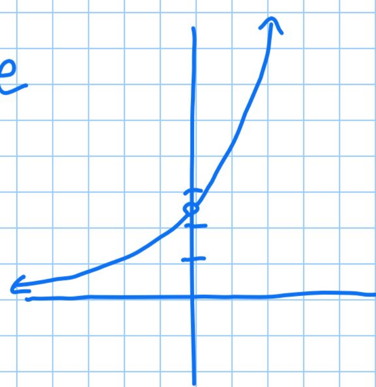
3) $2x^2+5x+3$ $\quad (2x+3)(x+1)$

4) $x^2+11x+10$ $\quad (x+1)(x+10)$

5) $3x^2-7x-10$ $\quad (3x-10)(x+1)$

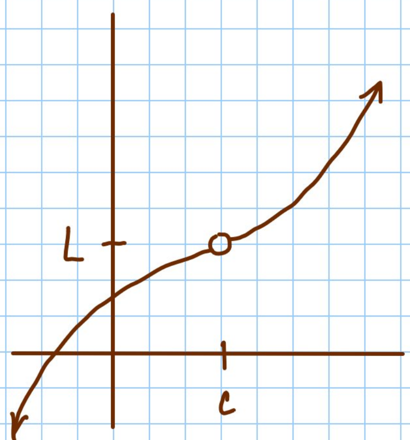
8) $\lim\limits_{h\to 0} \dfrac{e^{1+h}-e}{h} \approx 2.72 = e$

graph $y=\dfrac{e^{1+x}-e}{x}$



Section 2.2 Limits

Consider the graph of $f(x)$:



We say $\lim\limits_{x\to c} f(x) = L$

limit as x approaches c of $f(x)$ is L.

y is getting close to L as x gets close to c.

Note that $f(c) \neq L$ since there's

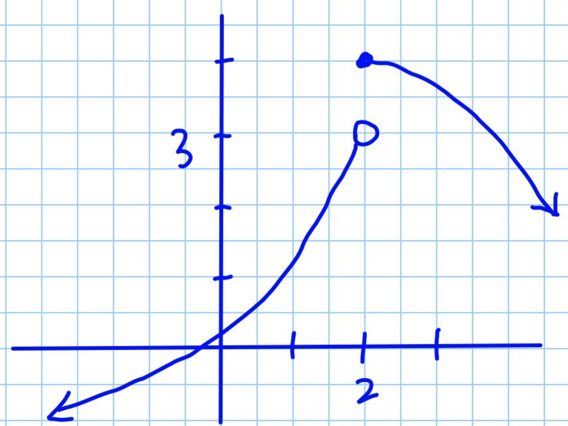
a hole in the graph.

We can use a table to evaluate a limit

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

x	-.03	-.02	-.01	0	.01	.02	.03
y	.99985	.99993	.99998	ERROR	.99998	.99993	.99985

Left-hand and Right-hand Limits



$$\lim_{x \to 2^-} f(x) = 3$$

limit of f(x) as x approaches 2 from the left

$$\lim_{x \to 2^+} f(x) = 4$$

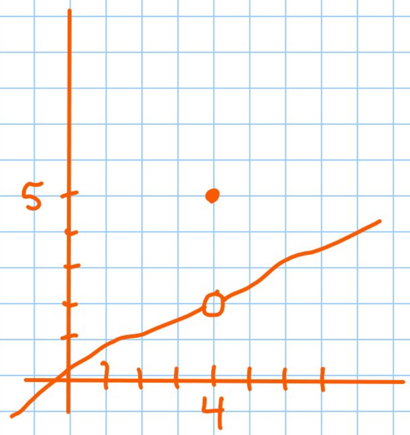
limit as x approaches from the right.

In this case $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

We say $\underbrace{\lim_{x \to 2} f(x)}$ does not exist

2-sided limit $\Rightarrow$ to exist, left-hand and right-hand limits must exist and be equal.

ex:



$$\lim_{x\to 4^-} f(x) = 2$$

$$\lim_{x\to 4^+} f(x) = 2$$

$$\lim_{x\to 4} f(x) = 2$$

$$f(4) = 5$$

Theorem 1.2 Properties of Limits

1) If b is a constant, $\lim\limits_{x\to c} \left(b\, f(x)\right) = b\left(\lim\limits_{x\to c} f(x)\right)$

2) $\lim\limits_{x\to c} \left(f(x) \pm g(x)\right) = \lim\limits_{x\to c} f(x) \pm \lim\limits_{x\to c} g(x)$

3) $\lim\limits_{x\to c} \left(f(x)\cdot g(x)\right) = \lim\limits_{x\to c} f(x) \cdot \lim\limits_{x\to c} g(x)$

4) $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$

5) $\lim\limits_{x\to c} K = K$

6) $\lim\limits_{x\to c} x = c$

ex: $$\lim_{x\to 4} \frac{x^2-3}{2x+1} = \frac{\lim\limits_{x\to 4} (x^2-3)}{\lim\limits_{x\to 4} (2x+1)} = \frac{\lim\limits_{x\to 4} x^2 - \lim\limits_{x\to 4} 3}{\lim\limits_{x\to 4} 2x + \lim\limits_{x\to 4} 1}$$

$$= \frac{\lim\limits_{x\to 4} x \cdot \lim\limits_{x\to 4} x - 3}{2\lim\limits_{x\to 4} x + 1}$$

$$= \frac{4 \cdot 4 - 3}{2 \cdot 4 + 1} = \frac{13}{9}$$

So the properties allow us to substitute c in $x \to c$ in for x as long as $f(c)$ is defined.

ex: $\lim_{x \to 5} \frac{x^2 - 16}{x+4} = \frac{5^2 - 16}{5+4} = \frac{9}{9} = 1$

ex: $\lim_{x \to -4} \frac{x^2 - 16}{x+4} = \lim_{x \to -4} \frac{\cancel{(x+4)}(x-4)}{\cancel{x+4}} = \lim_{x \to -4} (x-4)$

$= -4 - 4$

$= -8$

p68-69

1, 2, 15, 16, 37, 38