Section 2.1 How Do We Measure Speed?

Consider the table which shows the height of a grapefruit at t seconds.

t (sec)	0	1	2	3	4	5	6
$s(t) = y$ (feet)	6	90	142	162	150	106	30

If $s(t)$ is the position of an object at a time t then the average velocity from $t=a$ to $t=b$ is

$$\frac{s(b)-s(a)}{b-a} = \frac{\text{change in position}}{\text{change in time}}$$

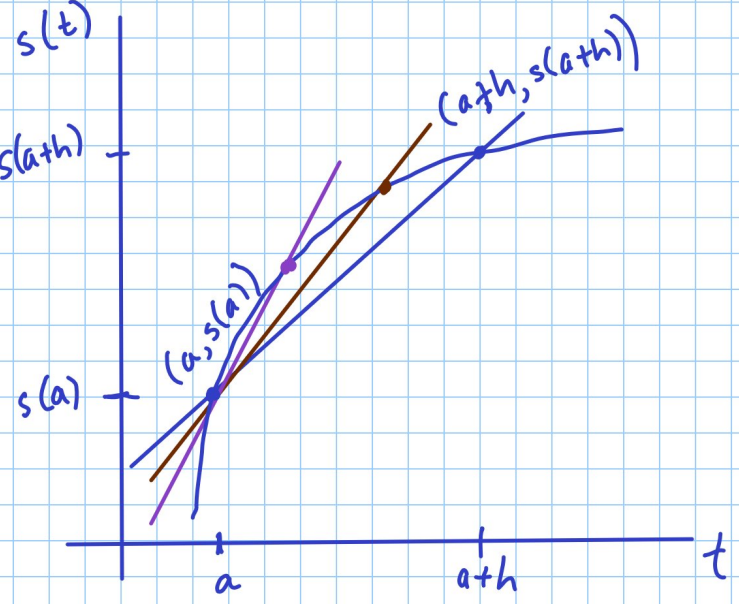
ex: Find the average velocity for $4 \le t \le 5$

$$\frac{s(5)-s(4)}{5-4} = \frac{106-150}{1} = -44 \text{ ft/sec}$$

ex: Find the average velocity for $1 \le t \le 3$

$$\frac{s(3)-s(1)}{3-1} = \frac{162-90}{3-1} = \frac{72}{2} = 36 \text{ ft/sec}$$

What if we wanted to find instantaneous velocity at $t=a$?



avg. velocity on $a \le t \le a+h$

$$\frac{s(a+h)-s(a)}{a+h-a}$$

$$\frac{s(a+h)-s(a)}{h}$$

slope of secant line

If we let $a+h$ approach a, then the average velocity gets closer to the instantaneous velocity.

We can say therefore that

$$\text{instantaneous velocity at } t=a = \lim_{h\to 0} \frac{s(a+h)-s(a)}{h}$$

As $h \to 0$ the secant line joining $(a, s(a))$ and $(a+h, s(a+h))$ approaches the tangent line at $(a, s(a))$

So instantaneous velocity is the slope of the tangent line

Slope of tan. line at $x=a$ means the same as slope of the curve at $x=a$.

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$s = 3t^2$

$t=1 \quad t=1+h$

i) $h=0.1$

$$\frac{s(1+h)-s(1)}{1+h-1} = \frac{s(1.1)-s(1)}{0.1}$$

$$= \frac{3(1.1)^2 - 3\cdot 1^2}{0.1}$$

$$= \frac{0.63}{0.1} = 6.3 \text{ m/sec}$$

ii) $h=0.01$

$$\frac{s(1.01)-s(1)}{.01} = \frac{3(1.01)^2 - 3\cdot 1^2}{.01}$$

$$= \frac{0.0603}{0.01} = 6.03 \text{ m/sec}$$

instantaneous velocity

$s(t) = 3t^2$

$$\lim_{h\to 0} \frac{s(1+h) - s(1)}{h} = \lim_{h\to 0} \frac{3(1+h)^2 - 3\cdot 1^2}{h}$$

$$= \lim_{h\to 0} \frac{3(1+2h+h^2) - 3}{h}$$

$$= \lim_{h\to 0} \frac{3+6h+3h^2-3}{h}$$

$$= \lim_{h\to 0} \frac{h(6+3h)}{h}$$

$$= 6 + 3\cdot 0$$

$$= 6$$

lim Lim

$$(1+h)^2 = (1+h)(1+h)$$

$$= 1+h+h+h^2 = 1+2h+h^2$$

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1, 3-5, 8, 13-16