

$$\lim_{x \rightarrow 0} \frac{(\sqrt{10+x} - \sqrt{10})(\sqrt{10+x} + \sqrt{10})}{x(\sqrt{10+x} + \sqrt{10})}$$

$$\lim_{x \rightarrow 0} \frac{10+x-10}{x(\sqrt{10+x} + \sqrt{10})}$$

$$\lim_{x \rightarrow 0} \frac{1}{\sqrt{10+x} + \sqrt{10}}$$

$$\frac{1}{\sqrt{10+0} + \sqrt{10}} = \frac{1}{\sqrt{10} + \sqrt{10}} = \frac{1}{2\sqrt{10}}$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{7+h} - \frac{1}{7}}{h} = \lim_{h \rightarrow 0} \frac{7 - (7+h)}{(7+h)7} \cdot \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{(7+h)7}$$

$$= \frac{-1}{(7+0)7} = -\frac{1}{49}$$

$$8) \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{x^3 + 3x^2 \Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$$

$$(x+\Delta x)(x+\Delta x)(x+\Delta x)$$

$$(x+\Delta x)(x^2 + 2x\Delta x + (\Delta x)^2)$$

$$x^3 + 2x^2\Delta x + x(\Delta x)^2 + x^2\Delta x + 2x(\Delta x)^2 + (\Delta x)^3$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\Delta x (3x^2 + 3x\Delta x + (\Delta x)^2)}{\Delta x}$$

$$= 3x^2 + 3x \cdot 0 + 0^2$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 = 3x^2$$

$$\begin{aligned}
 10) \lim_{h \rightarrow 0} \frac{\frac{1}{(10+h)^2} - \frac{1}{100}}{\frac{1}{h}} &= \lim_{h \rightarrow 0} \frac{100 - (10+h)^2}{(10+h)^2 \cdot 100} \cdot \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{100 - (100 + 20h + h^2)}{(10+h)^2 \cdot 100h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{100} - \cancel{100} - 20h - h^2}{(10+h)^2 \cdot 100h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{h}(-20-h)}{(10+h)^2 \cdot 100\cancel{h}} \\
 &= \frac{-20-0}{(10+0)^2 \cdot 100} = \frac{-20}{100 \cdot 100} = -\frac{1}{500}
 \end{aligned}$$

$$\lim_{h \rightarrow 0} \frac{(\sqrt{7+h} - \sqrt{7})(\sqrt{7+h} + \sqrt{7})}{h(\sqrt{7+h} + \sqrt{7})}$$

$$\sqrt{N} \cdot \sqrt{N} = N$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{(7+h)} - 7}{h(\sqrt{7+h} + \sqrt{7})}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}}{h(\sqrt{7+h} + \sqrt{7})} = \frac{1}{\sqrt{7+0} + \sqrt{7}} = \frac{1}{2\sqrt{7}}$$

$$\lim_{x \rightarrow 0} \frac{\left(\frac{1}{11+x} - \frac{1}{11} \right) (11+x)11}{x(11+x)11} = \lim_{x \rightarrow 0} \frac{\frac{1}{11+x} \cdot \frac{(11+x) \cdot 11}{1} - \frac{1}{11} \cdot \frac{(11+x) \cdot 11}{1}}{x(11+x)11}$$

$$\text{LCD} = (11+x)11$$

$$= \lim_{x \rightarrow 0} \frac{11 - (11+x)}{x(11+x)11}$$

$$= \lim_{x \rightarrow 0} \frac{\cancel{11}11 - x}{x(11+x) \cdot 11}$$

$$= \lim_{x \rightarrow 0} \frac{\cancel{x} - 1}{x(11+x) \cdot 11}$$

$$= \frac{-1}{(11+0)11} = -\frac{1}{121}$$

$$-\frac{5}{5} = -1$$

See LarsonCalculusforAP.com for an interactive version of this content.
To find the derivative of $f(x) = x^3 + 2x$, use the definition of the derivative as shown.

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} && \text{Definition of derivative} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^3 + 2(x + \Delta x) - (x^3 + 2x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + 2x + 2\Delta x - x^3 - 2x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + 2\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\cancel{\Delta x}[3x^2 + 3x\Delta x + (\Delta x)^2 + 2]}{\cancel{\Delta x}} \\ &= \lim_{\Delta x \rightarrow 0} [3x^2 + 3x\Delta x + (\Delta x)^2 + 2] \\ &= 3x^2 + 2 \end{aligned}$$

$$8) \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h} (3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= 3x^2 + 3x \cdot 0 + 0^2$$

$$= 3x^2$$

$$\begin{array}{cccc} & & 1 & \\ & & | & \\ & 1 & & 1 \\ & | & & | \\ 1 & 2 & & 1 \\ | & 3 & 3 & | \leftarrow \end{array}$$

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$