Section 8.5 The Dot Product

The product of 2 vectors is called the <u>dot product</u>

If $\mathbf{v} = a_1 i + b_1 j$ and $\mathbf{w} = a_2 i + b_2 j$ then

$$\mathbf{v} \cdot \mathbf{w} = a_1 a_2 + b_1 b_2$$

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$\mathbf{v} = 2i - 3j \qquad \mathbf{w} = 5i + 3j$

a) $\mathbf{v} \cdot \mathbf{w} = 2 \cdot 5 + (-3) \cdot 3 = 10 + (-9) = 1$

b) $\mathbf{w} \cdot \mathbf{v} = 5 \cdot 2 + 3 \cdot (-3) = 10 + (-9) = 1$

c) $\mathbf{v} \cdot \mathbf{v} = 2 \cdot 2 + (-3)(-3) = 4 + 9 = 13$

d) $\|\mathbf{v}\| = \sqrt{2^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13}$

Properties of the Dot Product

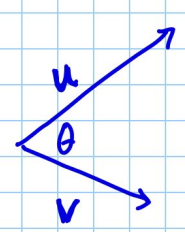
<u>Commutative:</u> $\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$

<u>Distributive:</u> $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

$$\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$$

$$\mathbf{0} \cdot \mathbf{v} = 0$$

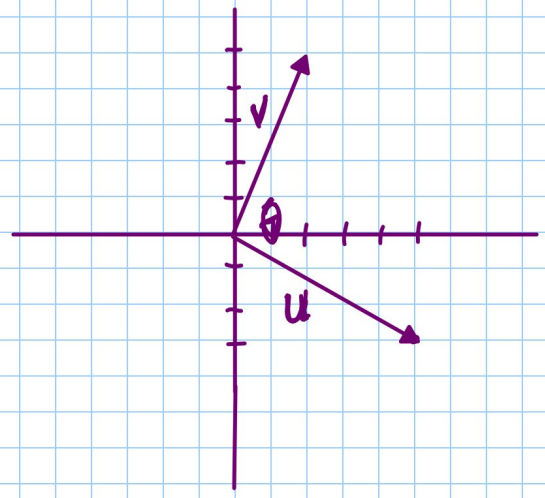
The Dot Product can be used to find the angle between 2 vectors



$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \cdot \|\mathbf{v}\|}$$

ex: $\mathbf{u} = 4i - 3j$

$\mathbf{v} = 2i + 5j$



$$u \cdot v = 4 \cdot 2 + (-3) \cdot 5 = 8 - 15 = -7$$

$$\|u\| = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$

$$\|v\| = \sqrt{2^2 + 5^2} = \sqrt{29}$$

$$\cos\theta = \frac{-7}{5\sqrt{29}}$$

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$$\theta = \cos^{-1}\left(-7/(5\sqrt{29})\right)$$

$$\theta = 105^\circ$$

v and **w** are <u>parallel</u> if there's a scalar α such that $\mathbf{v} = \alpha \mathbf{w}$

<u>ex</u>: $\mathbf{v} = 3i - j$ and $\mathbf{w} = 6i - 2j$ are parallel since $\mathbf{v} = \frac{1}{2}\mathbf{w}$

v and **w** are <u>orthogonal</u> (perpendicular) if $\mathbf{v} \cdot \mathbf{w} = 0$

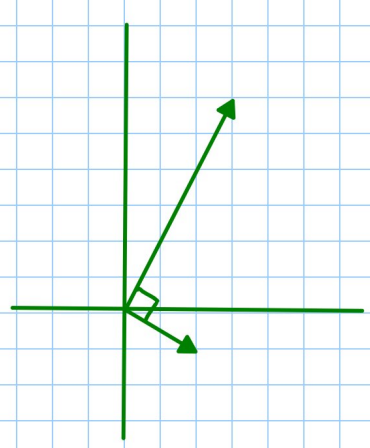
<u>ex</u>: $\mathbf{v} = 2i - j$ $\quad \mathbf{w} = 3i + 6j$

$$\mathbf{v} \cdot \mathbf{w} = 2 \cdot 3 + (-1) \cdot 6 = 0$$

$$\cos\theta = \frac{0}{\|\mathbf{v}\|\|\mathbf{w}\|} = 0$$

$$\theta = \cos^{-1} 0$$

$$\theta = 90^\circ$$



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