Math 112 Review for Last Test Part 1

Name: ______________________

Show all work.

$2\sin^2\theta - \sin\theta - 1 = 0$

Period $= \frac{2\pi}{3} = \frac{12\pi}{18}$

$2\cos(2\theta) = \sqrt{3}$

1. Solve each trig equation on $0 \le \theta < 2\pi$.

   a. $\sqrt{3}\tan^2\theta - \tan\theta = 0$

   $\tan\theta(\sqrt{3}\tan\theta - 1) = 0$

   $\tan\theta = 0 \quad \tan\theta = \frac{\sqrt{3}}{3}$

   $\theta = 0, \pi, \frac{\pi}{6}, \frac{7\pi}{6}$

   b. $2\sin(3\theta) = -1$

   $\sin(3\theta) = -\frac{1}{2}$

   $3\theta = \frac{7\pi}{6} \Rightarrow \theta = \frac{7\pi}{18}, \frac{19\pi}{18}, \frac{31\pi}{18}$

   $3\theta = \frac{11\pi}{6} \Rightarrow \theta = \frac{11\pi}{18}, \frac{23\pi}{18}, \frac{35\pi}{18}$

2. Find the exact value of each of the following:

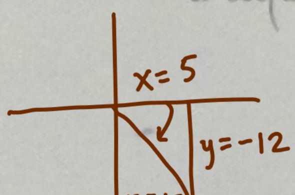
   a. $\cos(\tan^{-1}(-\sqrt{3}))$

   $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

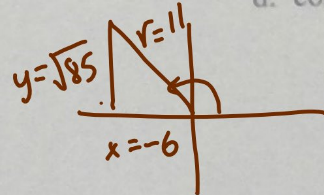
   b. $\sin^{-1}\left(\sin\frac{7\pi}{4}\right)$

   $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

   c. $\sec\left(\tan^{-1}\left(-\frac{12}{5}\right)\right) = \frac{13}{5}$



   d. $\cot\left(\cos^{-1}\left(-\frac{6}{11}\right)\right) = -\frac{6}{\sqrt{85}}$

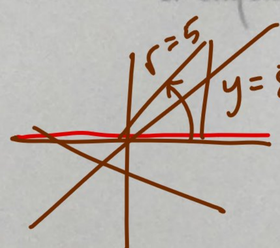


   $(-6)^2 + y^2 = 11^2$

   $36 + y^2 = 121$

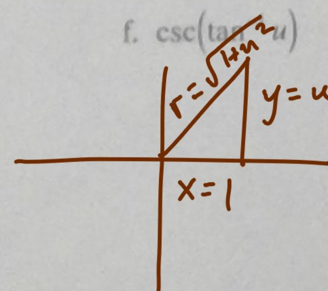
   $y^2 = 85$

   e. $\sin\left(\sin^{-1}\frac{8}{5}\right)$



   undefined

   f. $\csc(\tan^{-1}u) = \frac{\sqrt{1+u^2}}{u}$



3. Use the angle sum or difference formulas to find the exact value of:

   a. $\cos 165^\circ = \cos(135^\circ + 30^\circ) = \cos 135^\circ \cos 30^\circ - \sin 135^\circ \sin 30^\circ$

   $= -\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{-\sqrt{6} - \sqrt{2}}{4}$

   b. $\sin 255^\circ = \sin(225^\circ + 30^\circ) = \sin 225^\circ \cos 30^\circ + \cos 225^\circ \sin 30^\circ$

   $= -\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{2}}{2}\right)\cdot\frac{1}{2}$

   $= -\frac{\sqrt{6} - \sqrt{2}}{4}$

PPP 3) $\cos 195^\circ$

$\sin 345^\circ$

PPP 4) $\sin 22.5°$

· BONUS: use half-angle formula twice for $\cos 11.25°$

4. Use the half-angle formula to calculate $\cos 165°$. QII

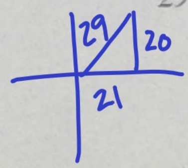
$$\cos 165° = \cos\frac{330°}{2} = -\sqrt{\frac{1+\cos 330°}{2}} = -\sqrt{\frac{\left(1+\frac{\sqrt{3}}{2}\right)}{2}\cdot\frac{2}{2}} = -\sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= -\frac{\sqrt{2+\sqrt{3}}}{2}$$

5. What is the decimal approximation to the nearest 1000th for the answers to 3a and 4?

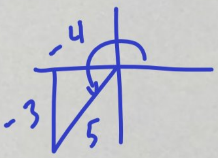
-0.966

6. Given $\sin\alpha = \frac{20}{29}$ with α in QI and $\csc\beta = -\frac{5}{3}$ with β in $QIII$, find each of the following:



$\sin\alpha = \frac{20}{29}$

$\cos\alpha = \frac{21}{29}$



$\sin\beta = -\frac{3}{5}$

$\cos\beta = -\frac{4}{5}$

a. $\sin(2\alpha)$

$$2\sin\alpha\cos\alpha = \frac{2}{1}\cdot\frac{20}{29}\cdot\frac{21}{29} = \frac{840}{841}$$

b. $\cos(2\alpha)$

$$\cos^2\alpha - \sin^2\alpha = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2 = \frac{441}{841} - \frac{400}{841} = \frac{41}{841}$$

c. $\tan(2\alpha)$

$$\frac{840}{41}$$

d. What quadrant is the angle 2α in? QI

e. $\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{\left(1-\frac{21}{29}\right)}{2}\cdot\frac{29}{29}} = \sqrt{\frac{29-21}{58}}$

$$= \sqrt{\frac{8}{58}} = \sqrt{\frac{4}{29}} = \frac{2}{\sqrt{29}}$$

f. $\tan\frac{\beta}{2} = \frac{1-\cos\beta}{\sin\beta} = \frac{1-\left(-\frac{4}{5}\right)}{-\frac{3}{5}} = \frac{\left(1+\frac{4}{5}\right)5}{-\frac{3}{5}\cdot 5} = \frac{5+4}{-3}$

$$= \frac{9}{-3} = -3$$

PPP

2) a) $\sec\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

e) $\cos\left(\cos^{-1}\left(-\frac{7}{6}\right)\right)$

b) $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

f) $\tan\left(\sin^{-1} u\right)$

c) $\csc\left(\cos^{-1}\left(-\frac{7}{25}\right)\right)$

d) $\sec\left(\sin^{-1}\left(-\frac{13}{18}\right)\right)$

PPP

6) $\tan\alpha = \frac{24}{7}$ α in QIII

$\csc\beta = \frac{13}{5}$ β in QI.

Find: a) $\sin(2\alpha)$

b) $\cos(2\alpha)$

c) $\tan(2\alpha)$

d) $\sin(\alpha - \beta)$

e) $\cos(\alpha - \beta)$

f) $\tan\frac{\beta}{2}$