WARMUP Establish the identity

$$\cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cos\beta$$

Section 6.5 Double-Angle Formulas; Half-Angle Formulas

Double-Angle Formulas

$$\sin(2\theta)=2\sin\theta\cos\theta$$

$$\cos(2\theta)=\cos^2\theta-\sin^2\theta$$

or $2\cos^2\theta-1$

or $1-2\sin^2\theta$

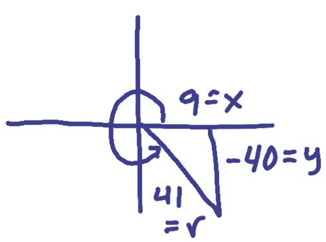
$$\tan(2\theta)=\frac{2\tan\theta}{1-\tan^2\theta}$$

Given $\sec\theta=\frac{41}{9}$ and θ is in QIV, find:

A) $\sin(2\theta)$ B) $\cos(2\theta)$ c) $\tan(2\theta)$

$\frac{r}{x}=\sec\theta$

3, 4, 5
5, 12, 13
7, 24, 25
8, 15, 17
9, 40, 41



$\sin\theta=-\frac{40}{41}$

$\cos\theta=\frac{9}{41}$

$\tan\theta=-\frac{40}{9}$

A) $\sin(2\theta)=2\sin\theta\cos\theta$

$=\frac{2}{1}\cdot\left(-\frac{40}{41}\right)\left(\frac{9}{41}\right)$

$=\frac{-720}{1681}\quad=\frac{y}{r}$

B) $\cos(2\theta)=\cos^2\theta-\sin^2\theta$

$=\left(\frac{9}{41}\right)^2-\left(-\frac{40}{41}\right)^2$

$=\frac{81}{1681}-\frac{1600}{1681}$

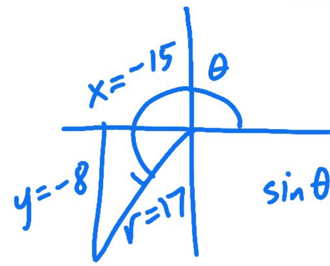
$=\frac{-1519}{1681}\quad=\frac{x}{r}$

For 2θ
$x=-1519$ $x<0$
$y=-720$ $y<0$
$r=1681$

c) $\tan(2\theta) = \frac{y}{x} = \frac{-720}{-1519} = \frac{720}{1519}$

What quadrant is 2θ in? Q III

Given $\cot\theta = \frac{15}{8}$ with θ in Q III, find



$\sin\theta = \frac{-8}{17}$

$\cos\theta = \frac{-15}{17}$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$= \frac{2}{1}\left(-\frac{8}{17}\right)\left(-\frac{15}{17}\right) = \frac{240}{289}$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= \left(-\frac{15}{17}\right)^2 - \left(-\frac{8}{17}\right)^2 = \frac{225}{289} - \frac{64}{289} = \frac{161}{289}$$

$$\tan(2\theta) = \frac{240}{161}$$

2θ is in Q I

2θ has
$x = 161$
$y = 240$
$r = 289$

$161^2 + 240^2 = 289^2$

Half-Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

You have to determine what quadrant $\frac{\theta}{2}$ is in and then choose the + or −.

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta}$$

Exact value of $\sin 112.5° = \sin\frac{225°}{2} = +\sqrt{\frac{1-\cos 225°}{2}}$

in Q II so sine is

$$= \sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$\frac{\sqrt{2}}{\cancel{2}}\cancel{2}$

$$= \sqrt{\frac{\left(1+\frac{\sqrt{2}}{2}\right)}{2}\cdot\frac{2}{2}}$$

$$= \sqrt{\frac{2+\sqrt{2}}{4}}$$

$$= \frac{\sqrt{2+\sqrt{2}}}{\sqrt{4}}$$

$$\boxed{\sin 112.5^\circ = \frac{\sqrt{2+\sqrt{2}}}{2}}$$

Use half-angle formula:

$$\cos 195^\circ = \cos \frac{390^\circ}{2} = -\sqrt{\frac{1+\cos 390^\circ}{2}}$$

Q III

so cosine is

$$= -\sqrt{\frac{1+\cos 30^\circ}{2}}$$

$$= -\sqrt{\frac{\left(1+\frac{\sqrt{3}}{2}\right)}{2} \cdot \frac{2}{2}}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$= -\sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= -\frac{\sqrt{2+\sqrt{3}}}{2}$$

Use half-angle for $\cos 105^\circ$

$\cos 105^\circ$ is in Q II

$$\cos 210^\circ = -\frac{\sqrt{3}}{2}$$

$$-\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2} \cdot \frac{2}{2}}$$

$$-\frac{\sqrt{2-\sqrt{3}}}{2}$$