

WARMUP

Copy these identities into your notes:

BASIC TRIG IDENTITIES

$$\underline{\text{Quotient}} : \tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\underline{\text{Reciprocal}} : \csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\underline{\text{Pythagorean}} : \sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta \Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\tan^2 \theta + 1 = \sec^2 \theta \Rightarrow \tan^2 \theta = \sec^2 \theta - 1 \Rightarrow \sec^2 \theta - \tan^2 \theta = 1$$

$$1 + \cot^2 \theta = \csc^2 \theta \Rightarrow \cot^2 \theta = \csc^2 \theta - 1 \Rightarrow \csc^2 \theta - \cot^2 \theta = 1$$

Section 6.3 Trig Identities

An identity is an equation that is true for any number.

$$\underline{\text{ex}} : (x+1)^2 = x^2 + 2x + 1$$

$$\underline{\text{ex}} : \sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta = (\sin \theta)^2$$

Establish the identity:

$$\underline{\text{ex}} : \sec \theta \cdot \sin \theta = \tan \theta$$

To do these problems, rewrite the more complicated side and use our basic trig identities to transform the expression into the other side.

$$\sec \theta \cdot \sin \theta = \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1}$$

$$= \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta$$

ex: $\sin \theta \cdot \csc \theta - \cos^2 \theta = \sin^2 \theta$

$$\sin \theta \cdot \csc \theta - \cos^2 \theta = \sin \theta \cdot \frac{1}{\sin \theta} - \cos^2 \theta$$

$$= 1 - \cos^2 \theta$$

$$= \sin^2 \theta$$

Assignment 1 Week 5

p480 1, 5, 6, 9, 13, 15, 16

1) $\csc \theta \cdot \cos \theta = \cot \theta$

$$\csc \theta \cdot \cos \theta = \frac{1}{\sin \theta} \cdot \frac{\cos \theta}{1}$$

$$= \frac{\cos \theta}{\sin \theta}$$

$$= \cot \theta$$

5) $\cos \theta (\tan \theta + \cot \theta) = \csc \theta$

$$\cos \theta (\tan \theta + \cot \theta) = \frac{\cos \theta}{1} \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)$$

$$= \frac{\sin \theta}{1} + \frac{\cos^2 \theta}{\sin \theta} = \frac{\sin \theta}{\sin \theta} \cdot \frac{\sin \theta}{1} + \frac{\cos^2 \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta}$$

$$\frac{3}{3} \cdot \frac{5}{1} + \frac{1}{3}$$

$$\frac{15+1}{3} = \frac{1}{\sin \theta} = \csc \theta$$

$$13) \cos^2 \theta (1 + \tan^2 \theta) = 1$$

$$\begin{aligned} \cos^2 \theta (1 + \tan^2 \theta) &= \cos^2 \theta \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta}\right) \\ &= \cos^2 \theta + \cancel{\frac{\cos^2 \theta}{1} \cdot \frac{\sin^2 \theta}{\cos^2 \theta}} \\ &= \cos^2 \theta + \sin^2 \theta \\ &= 1 \end{aligned}$$

$$15) (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$$

$$\begin{aligned} (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 &= (\underbrace{\sin \theta + \cos \theta}_{\sin^2 \theta + 2\sin \theta \cos \theta + \cos^2 \theta} \underbrace{(\sin \theta + \cos \theta)}_{\sin^2 \theta - 2\sin \theta \cos \theta + \cos^2 \theta}) \\ &= \sin^2 \theta + 2\sin \theta \cos \theta + \cos^2 \theta \\ &\quad + \sin^2 \theta - 2\sin \theta \cos \theta + \cos^2 \theta \end{aligned}$$

$$= \underline{1} + \underline{1}$$

$$= 2$$