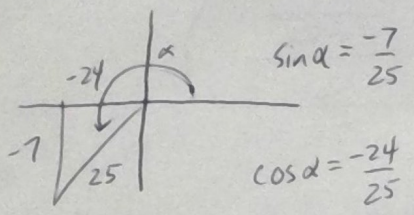
Precalculus Practice Test 2

Name: Key

Complete each of the following problems. Show all necessary work.

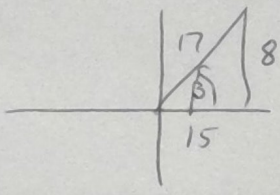
In 1-4, use the following information:

$\sec\alpha = -\frac{25}{24}$ with $\pi < \alpha < \frac{3\pi}{2}$ $\qquad$ $\tan\beta = \frac{8}{15}$ with $0 < \beta < \frac{\pi}{2}$



$\sin\alpha = \frac{-7}{25}$

$\cos\alpha = \frac{-24}{25}$



$\sin\beta = \frac{8}{17}$

$\cos\beta = \frac{15}{17}$

1. Find the exact value of $\sin(\alpha - \beta)$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta = -\frac{7}{25}\cdot\frac{15}{17} - \left(\frac{-24}{25}\right)\frac{8}{17} = \frac{-105}{425} + \frac{192}{425}$

$= \frac{87}{425} = \frac{y}{r}$

2. Find the exact value of $\cos(\alpha - \beta)$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{-24}{25}\cdot\frac{15}{17} + \frac{-7}{25}\cdot\frac{8}{17} = \frac{-360-56}{425}$

$= \frac{-416}{425} = \frac{x}{r}$

3. Find the exact value of $\tan(\alpha - \beta)$

$= -\frac{87}{416} = \frac{y}{x}$

4. In which quadrant is the angle $\alpha - \beta$?

$y > 0$ $\quad x < 0$

QII

PPP

$\cot\alpha = \frac{40}{9}$ $\quad \pi < \alpha < \frac{3\pi}{2}$

$\sec\beta = \frac{13}{5}$ $\quad 0 < \beta < \frac{\pi}{2}$

$\sin(\alpha+\beta)$ $\cos(\alpha+\beta)$ $\tan(\alpha+\beta)$ what quadrant $\alpha+\beta$ in?

In 5 and 6 use the half-angle formulas to calculate the exact value of each of the following:

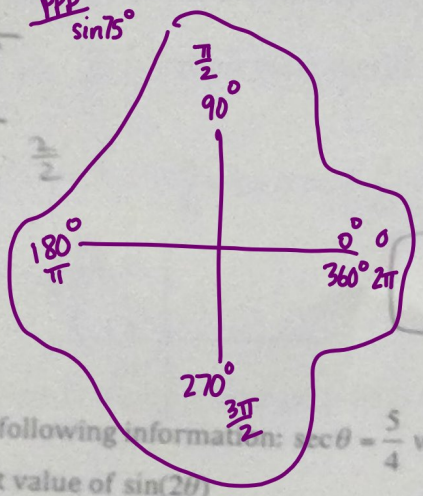
5. sin15°

PPP sin75°

$\sin 15° = \sqrt{\frac{1-\cos 30°}{2}}$

$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$ $\frac{2}{2}$

$= \frac{\sqrt{2-\sqrt{3}}}{2}$



6. cos157.5°

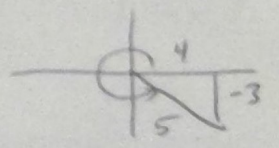
$= -\sqrt{\frac{1+\cos 315°}{2}}$

$= -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$ $\frac{2}{2}$

$= \frac{-\sqrt{2+\sqrt{2}}}{2}$

PPP cos 112.5°

In 7-10, use the following information: $\sec\theta = \frac{5}{4}$ with θ in *QIV*



$\sin\theta = -\frac{3}{5}$

$\cos\theta = \frac{4}{5}$

7. Find the exact value of sin(2θ)

$\sin(2\theta) = 2\sin\theta\cos\theta$

$= 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{-24}{25} = \frac{y}{r}$

8. Find the exact value of cos(2θ)

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25} = \frac{x}{r}$

PPP $\cot\theta = \frac{9}{40}$ θ in QIII

sin(2θ), cos(2θ), tan(2θ)

$\tan\frac{\theta}{2}$

9. Find the exact value of tan(2θ)

$= -\frac{24}{7} = \frac{y}{x}$

10. Find the exact value of $\tan\frac{\theta}{2}$

$= \frac{1-\cos\theta}{\sin\theta} = \frac{\left(1-\frac{4}{5}\right)}{-\frac{3}{5}}\cdot\frac{5}{5} = \frac{5-4}{-3}$

$= -\frac{1}{3}$

In 11-13, establish each identity:

11. $\cos\theta\sec\theta - \sin^2\theta = \cos^2\theta$

$\cos\theta\sec\theta - \sin^2\theta = \cos\theta \cdot \frac{1}{\cos\theta} - \sin^2\theta$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

12. $\frac{\sec\theta}{1+\sec\theta} = \frac{1-\cos\theta}{\sin^2\theta}$

$\frac{\sec\theta}{1+\sec\theta} = \frac{\frac{1}{\cos\theta}}{\left(1+\frac{1}{\cos\theta}\right)} \cdot \frac{\cos\theta}{\cos\theta} = \frac{1}{\cos\theta+1} \cdot \frac{(1-\cos\theta)}{(1-\cos\theta)}$

$= \frac{1-\cos\theta}{1-\cos^2\theta}$

$= \frac{1-\cos\theta}{\sin^2\theta}$

PPP

$\frac{\cos\theta+\sin\theta-\sin^3\theta}{\sin\theta} = \cot\theta + \cos^2\theta$

$\frac{\sin\theta+\cos\theta-\cos^3\theta}{\cos\theta} = \tan\theta + \sin^2\theta$

13. $\sin(\alpha+\beta)+\sin(\alpha-\beta) = 2\sin\alpha\cos\beta$

$\sin(\alpha+\beta) + \sin(\alpha-\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= 2\sin\alpha\cos\beta$

14. Use your chart to find the exact value of each of the following:

a. $\sin\frac{25\pi}{6} = \frac{1}{2}$

$\frac{25\pi}{6}\cdot\frac{180}{\pi} = 750°$ (30)

$\cos\frac{31\pi}{6} = \cos 210° = -\frac{\sqrt{3}}{2}$

b. $\cos 495° = -\frac{\sqrt{2}}{2}$

$\frac{17\pi}{2}\cdot\frac{180}{\pi} = 1530°$ (90)

-720

810

-720

90

c. $\tan(-315°) = 1$

d. $\cot\frac{17\pi}{2} = 0$

e. $\sec 630° = \text{und}$

$\frac{31\pi}{6}\cdot\frac{180}{\pi} = 930°$ (30)

-360

210

-320

$\sec 270°$

f. $\csc\frac{16\pi}{3} = -\frac{2\sqrt{3}}{3}$

15. The terminal side of θ passes through $(-6,11)$. Find the exact values of the six trig ratios. (x, y)

$\sin\theta =$ _____

$\cos\theta =$ _____

$\tan\theta =$ _____

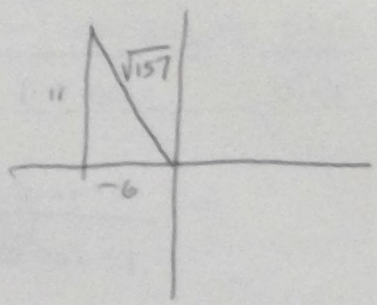
$\csc\theta =$ _____

$\sec\theta =$ _____

$\cot\theta =$ _____

$(-6)^2 + 11^2 = 157$

$r = \sqrt{157}$



121
36
157

1
13)15 7

$\sin\theta = \frac{11\sqrt{157}}{157}$ $\quad \csc\theta = \frac{\sqrt{157}}{11}$

$\cos\theta = -\frac{6\sqrt{157}}{157}$ $\quad \sec\theta = -\frac{\sqrt{157}}{6}$

$\tan\theta = -\frac{11}{6}$ $\quad \cot\theta = -\frac{6}{11}$

PPP

$\cos\frac{17\pi}{4}$ $\quad \cot\left(\frac{19\pi}{3}\right)$

$\sin(-750°)$ $\quad \sec 405°$

$\tan\left(\frac{29\pi}{6}\right)$ $\quad \csc(-930°)$

PPP terminal side $(6,-7)$
Find six trig ratios

3, 4, 5
5, 12, 13
7, 24, 25
8, 15, 17
9, 40, 41

Note Sheet

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

II $(x<0, y>0)$	I $(x>0, y>0)$
$\sin\theta$, $\csc\theta$ + $\tan\theta$, $\cot\theta$, $\cos\theta$, $\sec\theta$ } −	All trig functions +
$\tan\theta$, $\cot\theta$ + $\sin\theta$, $\csc\theta$, $\cos\theta$, $\sec\theta$ } −	$\cos\theta$, $\sec\theta$ + $\sin\theta$, $\csc\theta$, $\tan\theta$, $\cot\theta$ } −
III $(x<0, y<0)$	IV $(x>0, y<0)$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $\quad \tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin(-\theta) = -\sin\theta$ $\quad \csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$ $\quad \sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\quad \cot(-\theta) = -\cot\theta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$