

Assignment 5/10/17

Given $\tan \alpha = \frac{3}{4}$ with α in QI and $\csc \beta = -\frac{13}{12}$ with β in QIV, find:

1) $\sin(\alpha - \beta)$

5) $\sin(2\alpha)$

9) $\sin\left(\frac{\beta}{2}\right)$

2) $\cos(\alpha - \beta)$

6) $\cos(2\alpha)$

10) $\tan\left(\frac{\beta}{2}\right)$

3) $\tan(\alpha - \beta)$

7) $\tan(2\alpha)$

4) $\alpha - \beta$ is in Q__?

8) 2α is in Q__?

Use half-angle formulas to find:

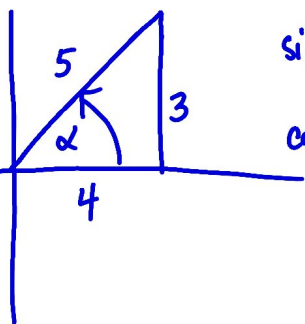
11) $\sin 67.5^\circ$

12) $\tan 165^\circ$

$$\sin \theta = \frac{y}{r} \quad \csc \theta = \frac{r}{y}$$

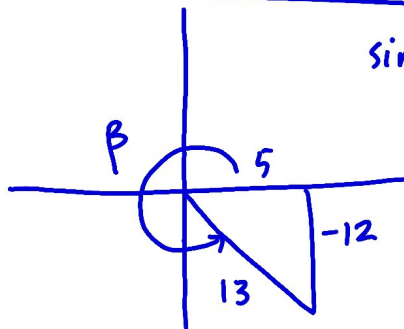
$$\cos \theta = \frac{x}{r} \quad \sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x} \quad \cot \theta = \frac{x}{y}$$



$$\sin \alpha = \frac{3}{5}$$

$$\cos \alpha = \frac{4}{5}$$



$$\sin \beta = -\frac{12}{13}$$

$$\cos \beta = \frac{5}{13}$$

$$1) \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta = \frac{3}{5} \cdot \frac{5}{13} - \frac{4}{5} \cdot \left(-\frac{12}{13}\right) = \frac{15}{65} + \frac{48}{65} = \frac{63}{65} \quad \frac{y}{r}$$

$$2) \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta = \frac{4}{5} \cdot \frac{5}{13} + \frac{3}{5} \cdot \left(-\frac{12}{13}\right) = \frac{20}{65} - \frac{36}{65} = \frac{-16}{65} \quad \frac{x}{r}$$

$$3) \tan(\alpha - \beta) = \frac{63}{-16}$$

4) $\alpha - \beta$ is in QII since $x < 0, y > 0$

$$5) \sin(2\alpha) = 2 \sin \alpha \cos \alpha = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$$

$$6) \cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$7) \tan(2\alpha) = \frac{24}{7}$$

8) 2α is in QI since $x > 0, y > 0$

$$9) \sin \frac{\beta}{2} = + \sqrt{\frac{1 - \cos \beta}{2}} = \sqrt{\frac{(1 - \frac{5}{13}) \cdot \frac{13}{13}}{2}} = \sqrt{\frac{13 - 5}{26}} = \sqrt{\frac{8}{26}}$$

β is between 270° and 360° (Q IV)

so $\frac{\beta}{2}$ is between 135° and 180° (Q II)

$$= \sqrt{\frac{4}{13}}$$

$$= \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$$

$$= \frac{2\sqrt{13}}{13}$$

$$10) \tan \frac{\beta}{2} = \frac{1 - \cos \beta}{\sin \beta} = \frac{(1 - \frac{5}{13}) \cdot 13}{(-\frac{12}{13}) \cdot 13}$$

$$= \frac{13 - 5}{-12}$$

$$= \frac{8}{-12}$$

$$= -\frac{2}{3}$$

$$11) \sin 67.5^\circ = \sin \frac{135^\circ}{2} = + \sqrt{\frac{1 - \cos 135^\circ}{2}} = \sqrt{\frac{1 - (-\frac{\sqrt{2}}{2})}{2}} = \sqrt{\frac{(1 + \frac{\sqrt{2}}{2}) \cdot 2}{2} \cdot \frac{2}{2}}$$

$$= \sqrt{\frac{2 + \sqrt{2}}{4}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$12) \tan 165^\circ = \tan \frac{330^\circ}{2} = \frac{1 - \cos 330^\circ}{\sin 330^\circ} = \frac{(1 - \frac{\sqrt{3}}{2}) \cdot 2}{(-\frac{1}{2}) \cdot 2} = \frac{2 - \sqrt{3}}{-1} = -2 + \sqrt{3}$$