

## Section 8.5 Calculus and Economics Applications

### Definitions:

The future value,  $\$B$ , of a payment,  $\$P$ , is the amount to which the  $\$P$  would grow if deposited in an interest bearing account.

ex: if we deposit  $\$100$  in an account earning 7% annual interest the future value after 1 year is  $\$107$

The present value,  $\$P$  of a future payment  $\$B$  is the amount that would have to be deposited today to produce  $\$B$  in an indicated amount of time.

ex: if we're saving for a  $\$300$  game system we want to buy one year from now and we earn 10% annual interest, the present value is  $\$272.73$

$$\begin{aligned} .10P + P &= 300 \\ 1.1P &= 300 \\ P &= \$272.73 \end{aligned}$$

FORMULAS:  $r =$  annual rate of interest

$t =$  time in years

ANNUAL COMPOUNDING:  $B = P(1+r)^t$

$$P = \frac{B}{(1+r)^t}$$

CONTINUOUS COMPOUNDING:  $B = Pe^{rt}$

$$P = \frac{B}{e^{rt}} = Be^{-rt}$$

ex 1 p 378

$\$1,000,000$  in 4  $\$250,000$  payments vs.

$\$920,000$  Now

Present Value =  $\$920,000$

1:  $\$250,000$

2:  $P = 250000e^{-.06 \cdot 1} = 235,441.13$

3:  $P = 250000e^{-.06 \cdot 2} = 221,730.11$

4:  $P = 250000e^{-.06 \cdot 3} = 208,817.55$

Present Value =  $\$915,988.79$

# Using future values

4 250,000 deposit

$$1: 250,000e^{.06 \cdot 3} = 299,304.34$$

$$2: 250,000e^{.06 \cdot 2} = 281,874.21$$

$$3: 250,000e^{.06 \cdot 1} = 265,459.14$$

$$4: 250,000 = 250,000$$

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$$1,096,637.69$$

920,000 Now

$$B = 920,000e^{.06 \cdot 3}$$

$$B = \$1,101,439.97$$

of \$2000 a year, for a period of 5 years, if the continuous interest rate is 8%.

## Problems

5. Draw a graph, with time in years on the horizontal axis, of what an income stream might look like for a company that sells sunscreen in the northeast United States.
6. On April 15, 1999, Maria Grasso won the largest lottery amount ever awarded up to that date. She was given her choice between \$197 million, paid out continuously over 26 years, or a lump sum of \$104 million, paid immediately.
  - (a) Which option is better if the interest rate is 6%, compounded continuously? An interest rate of 5%?
  - (b) The winner chose the lump sum option. What assumption was she making about interest rates?
7. (a) A bank account earns 10% interest compounded continuously. At what (constant, continuous) rate must a parent deposit money into such an account in order to save \$100,000 in 10 years for a child's college expenses?
  - (b) If the parent decides instead to deposit a lump sum now in order to attain the goal of \$100,000 in 10 years, how much must be deposited now?
8. (a) If you deposit money continuously at a constant rate of \$1000 per year into a bank account that earns 5% interest, how many years will it take for the balance to reach \$10,000?
  - (b) How many years would it take if the account had \$2000 in it initially?

9. A business associate who owes you \$3000 offers to pay you \$2800 now, or else pay you three yearly installments of \$1000 each, with the first installment paid now. If you

use only financial reasons to make your decision, which option should you choose? Justify your answer, assuming a 6% market interest rate, compounded continuously.

10. Big Tree McGee is negotiating his rookie contract with a professional basketball team. They have agreed to a three-year deal which will pay Big Tree a fixed amount at the end of each of the three years, plus a signing bonus at the beginning of his first year. They are still haggling about the amounts and Big Tree must decide between a big signing bonus and fixed payments per year, or a smaller bonus with payments increasing each year. The two options are summarized in the table. All values are payments in millions of dollars.

	Signing bonus	Year 1	Year 2	Year 3
Option #1	6.0	2.0	2.0	2.0
Option #2	1.0	2.0	4.0	6.0

- (a) Big Tree decides to invest all income in stock funds which he expects to grow at a rate of 10% per year, compounded continuously. He would like to choose the contract option which gives him the greater future value at the end of the three years when the last payment is made. Which option should he choose?
- (b) Calculate the present value of each contract offer.

11. Sales of Version 6.0 of a computer software package start high and decrease exponentially. At time  $t$ , in years, the sales are  $s(t) = 50e^{-t}$  thousands of dollars.

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