

WARMUP

You deposit \$100 into an account on 10/1, 11/1, and 12/1. You earn 9% compounded monthly. How much is in your account on 12/31?

$$FV = P(1+i)^n$$

$$\underline{10/1} \quad P=100 \quad n=3 \quad i = \frac{.09}{12}$$

$$FV = 100 \left(1 + \frac{.09}{12}\right)^3 = \$102.27$$

$$\underline{11/1} \quad P=100 \quad n=2 \quad i = \frac{.09}{12}$$

$$FV = 100 \left(1 + \frac{.09}{12}\right)^2 = \$101.51$$

$$\underline{12/1} \quad P=100 \quad n=1 \quad i = \frac{.09}{12}$$

$$FV = 100 \left(1 + \frac{.09}{12}\right)^1 = \$100.75$$

$$\text{Total} = \$304.53$$

Section 5.3 Annuities

An annuity is a sequence of equal, regular payments into an account where each payment earns compound interest.

payment period - time between payments

term - time from beginning of first payment until the end of the last payment.

An annuity is expired when the money can be withdrawn.

simple - payment period is same as compounding period

annuity due - payment due at beginning of time period

ordinary annuity - payment is due at the end of time period.

ORDINARY ANNUITY

$$FV(\text{ordinary}) = \text{pymt} \cdot \frac{(1+i)^n - 1}{i}$$

future value
of an ordinary
annuity

i = periodic interest rate

pymt = payment

n = # of compoundings

ANNUITY DUE

$$FV(\text{due}) = \text{pymt} \cdot \frac{(1+i)^n - 1}{i} \cdot (1+i)$$
$$= FV(\text{ord}) \cdot (1+i)$$

ex3p334

TDA
a
x
f
r
e
e
r
r
e
n
n
u
i
t
y

#200 monthly

pymt = \$200

$r = 8\frac{3}{4}\% = .0875$

12 compoundings per year

30 → 65

$n = 12 \cdot 35 = 420$

$t = 35$

$$FV(\text{ord}) = \text{pymt} \cdot \frac{(1+i)^n - 1}{i} = 200 \cdot \frac{\left(\left(1 + \frac{.0875}{12} \right)^{420} - 1 \right)}{\left(\frac{.0875}{12} \right)}$$

calculator $200 \left(\left(1 + .0875/12 \right)^{420} - 1 \right) / (.0875/12)$

\$552,539.96

How much interest did Jim earn?

$$552,539.96 - 200 \cdot 420 = \$468,539.96$$

What if it is an annuity due?

$$\begin{aligned}FV(\text{due}) &= FV(\text{ord}) \cdot (1+i) \\&= 552,539.96 \cdot \left(1 + \frac{.0875}{12}\right) \\&= \$556,568.90\end{aligned}$$

Ben puts \$50 out of each of his bimonthly paychecks into an ordinary annuity earning 6% interest. He is 18 now and can withdraw his money when he's 65.

a) Find the FV.

b) Find Ben's total contribution

c) Find total interest.