

Section 8.5 Continued

Revenue of a company can come in all the time, so we're going to consider an income stream. We use $P(t)$ dollars/year to represent the rate at which deposits are made.

We can find the present value and future value of these income streams.

Assuming a continuous compounding rate of r we have formulas:

$$\text{Present Value} = \int_0^M P(t) e^{-rt} dt \quad \text{dollars}$$

$$\text{Future Value} = \int_0^M P(t) e^{r(M-t)} dt \quad \text{dollars}$$

M is the time over which we're trying to find the total income.

ex 2 p380

$$P(t) = 1000 \text{ dollars/year}$$

$$M = 20$$

$$r = 0.10$$

$$\text{Present Value} = \int_0^{20} 1000 e^{-0.10t} dt = \$8646.65$$

calculator

$$\text{Future Value} = \int_0^{20} 1000 e^{0.10(20-t)} dt = \$63,890.58$$

Annuities - People save for retirement using annuities which are a series of equal deposits that earn interest.

ORDINARY ANNUITY

$$FV = \text{pymt} \cdot \frac{\left(\left(1 + \frac{r}{n} \right)^{(nt)} - 1 \right)}{\left(\frac{r}{n} \right)}$$

Assume # of compoundings per year = # of payments per year

where FV = future value

pymt = payment

r = avg. rate of interest

n = # of compoundings per year

t = # of years

ex: Dean saves for retirement by putting \$200 of each of his bimonthly paychecks in an account earning 7%. He starts when he's 26 and will retire at 65. How much will ^{he} have?

$$\text{pymt} = 200$$

$$r = .07$$

$$t = 65 - 26 = 39$$

$$n = 24$$

$$\begin{aligned} \text{\# of payments} \\ 39 \times 24 = 936 \end{aligned}$$

$$\begin{aligned} \text{Dean's investment} \\ = 936 \times 200 = \$187,200 \end{aligned}$$

$$FV = 200 \cdot \frac{\left(\left(1 + \frac{.07}{24} \right)^{(24 \cdot 39)} - 1 \right)}{\left(\frac{.07}{24} \right)} = \$978,657.07$$

$$\text{calculator } 200 \left(\left(1 + .07/24 \right)^{(24 \cdot 39)} - 1 \right) / (.07/24)$$

Exercises and Problems for Section 8.5

Exercises

1. Find the future value of an income stream of \$1000 per year, deposited into an account paying 8% interest, compounded continuously, over a 10-year period.
2. Find the present and future values of an income stream of \$3000 per year over a 15-year period, assuming a 6% annual interest rate compounded continuously.
3. Find the present and future values of an income stream of \$2000 a year, for a period of 5 years, if the continuous interest rate is 8%.
4. A person deposits money into a retirement account, which pays 7% interest compounded continuously, at a rate of \$1000 per year for 20 years. Calculate:
 - (a) The balance in the account at the end of the 20 years.
 - (b) The amount of money actually deposited into the account.
 - (c) The interest earned during the 20 years.