Section 8.5 Continued

Revenue of a company can come in all the time, so we're going to consider an <u>income stream</u>. We use $P(t)$ dollars/year to represent the rate at which deposits are made.

We can find the present value and future value of these income streams.

Assuming a continuous compounding rate of r we have formulas:

$$\text{Present Value} = \int_0^M P(t)e^{-rt}\,dt \quad \text{dollars}$$

$$\text{Future Value} = \int_0^M P(t)e^{r(M-t)}\,dt \quad \text{dollars}$$

M is the time over which we're trying to find the total income.

<u>ex 2 p380</u> $P(t) = 1000$ dollars/year

$M = 20$

$r = 0.10$

$$\text{Present Value} = \underbrace{\int_0^{20} 1000e^{-0.10t}\,dt}_{\text{calculator}} = \$8646.65$$

$$\text{Future Value} = \int_0^{20} 1000e^{0.10(20-t)}\,dt = \$63{,}890.58$$

Annuities - People save for retirement using annuities which are a series of equal deposits that earn interest.

ORDINARY ANNUITY $$FV = pymt \cdot \frac{\left(\left(1+\frac{r}{n}\right)^{(nt)} - 1\right)}{\left(\frac{r}{n}\right)}$$

Assume # of compoundings per year = # of payments per year

where FV = future value
pymt = payment
r = avg. rate of interest
n = # of compoundings per year
t = # of years

ex: Dean saves for retirement by putting \$200 of each of his bimonthly paychecks in an account earning 7%. He starts when he's 26 and will retire at 65. How much will he have?

pymt = 200
r = .07
t = 65 − 26 = 39
n = 24

# of payments
39 × 24 = 936

Dean's investment
= 936 × 200 = \$187,200

$$FV = 200 \cdot \frac{\left(\left(1+\frac{.07}{24}\right)^{(24\cdot 39)} - 1\right)}{\left(\frac{.07}{24}\right)} = \$978,657.07$$

calculator 200((1+.07/24)^(24·39)−1)/(.07/24)

Exercises and Problems for Section 8.5

Exercises

1. Find the future value of an income stream of $1000 per year, deposited into an account paying 8% interest, compounded continuously, over a 10-year period.
2. Find the present and future values of an income stream of $3000 per year over a 15-year period, assuming a 6% annual interest rate compounded continuously.
3. Find the present and future values of an income stream of $2000 a year, for a period of 5 years, if the continuous interest rate is 8%.
4. A person deposits money into a retirement account, which pays 7% interest compounded continuously, at a rate of $1000 per year for 20 years. Calculate:
   (a) The balance in the account at the end of the 20 years.
   (b) The amount of money actually deposited into the account.
   (c) The interest earned during the 20 years.

2) $P(t)=3000$, $M=15$, $r=.06$

Present Value $= \int_0^{15} 3000e^{-.06t}\,dt = 29{,}671.52$

Future Value $= \int_0^{15} 3000e^{.06(15-t)}\,dt = 72{,}980.16$

4) $P(t)=1000$, $M=20$, $r=.07$

a) $FV = \int_0^{20} 1000e^{.07(20-t)}\,dt = \$43{,}645.71$

b) \$20000

c) $43{,}645.71 - 20{,}000 = \$23{,}645.71$

Extra Credit #7 due Tuesday March 14

1) A rock is thrown upward from a 96-ft high cliff at 80 ft/sec. Write functions for $a(t)$, $v(t)$, $h(t)$

a) How long does it take for the rock to reach its highest height?

b) What is the highest height?

c) How long does it take for the rock to hit the ground below the cliff?

d) with what velocity does it hit the ground?

2) Partial Fractions:

$$\int \frac{3}{x^2+6x+5}\,dx$$

3) Tabular:

$$\int (5x^2+3x)e^{4x}\,dx$$

4) Trig Substitution:

$$\int \frac{x}{\sqrt{49+x^2}}\,dx$$

5) Blue Sheet:

$$\int e^{-3x}\sin(7x)\,dx$$